Towards Heterotic Moduli Space Metrics

Jock McOrist

DAMTP, University of Cambridge

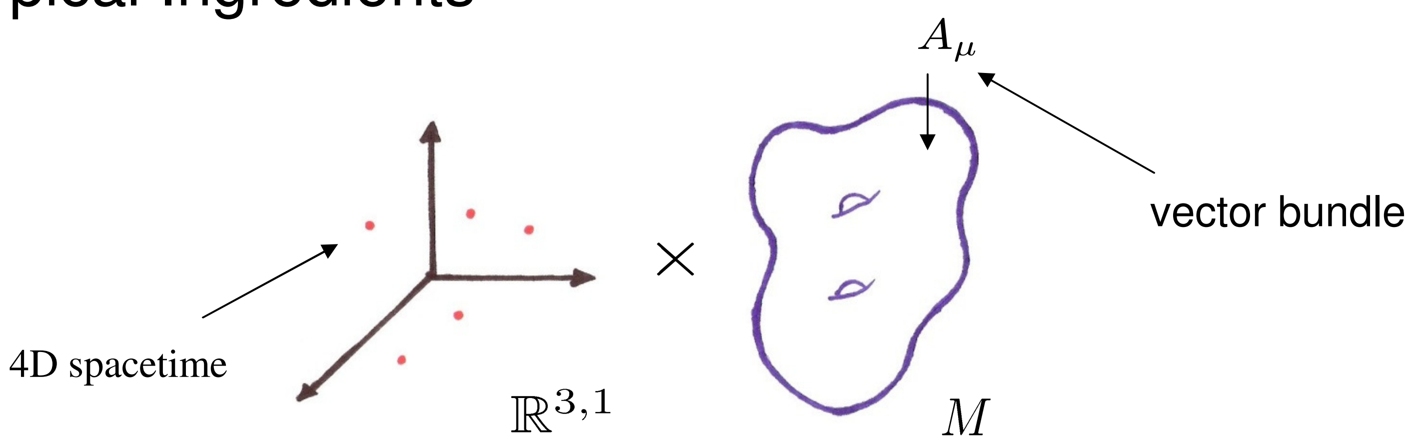
with Philip Candelas and Xenia de la Ossa

Why heterotic?

- Compactifications useful route to N=1, d=4 physics
- Heterotic interesting
  - Phenomenology: “Easy” route to quasi-realistic N=1 chiral gauge theories. No need for O-planes, branes, etc.
  - Geometry: framework for developing maths of vector bundles
  - Excellent framework for understanding quantum effects

What is a heterotic compactification?

- Typical Ingredients



- Geometry needs to solve SUGRA EOM:

$$R_{\mu\nu} = 0, \quad F_{0,2} = F_{2,0} = 0, \quad g^{\mu\bar{\nu}} F_{\mu\bar{\nu}} = 0$$

  and Bianchi $\quad dH = \frac{\alpha'}{4}(\mathrm{Tr} R \wedge R - \mathrm{Tr} F \wedge F)$

- Dimensional reduction gives a d=4 EFT:

$$\mathcal{L} = \int d^4\theta K(\Phi, \bar{\Phi}) + \int d^2\theta W(\Phi) + \mathrm{h.c.}$$

Geometries within Geometries

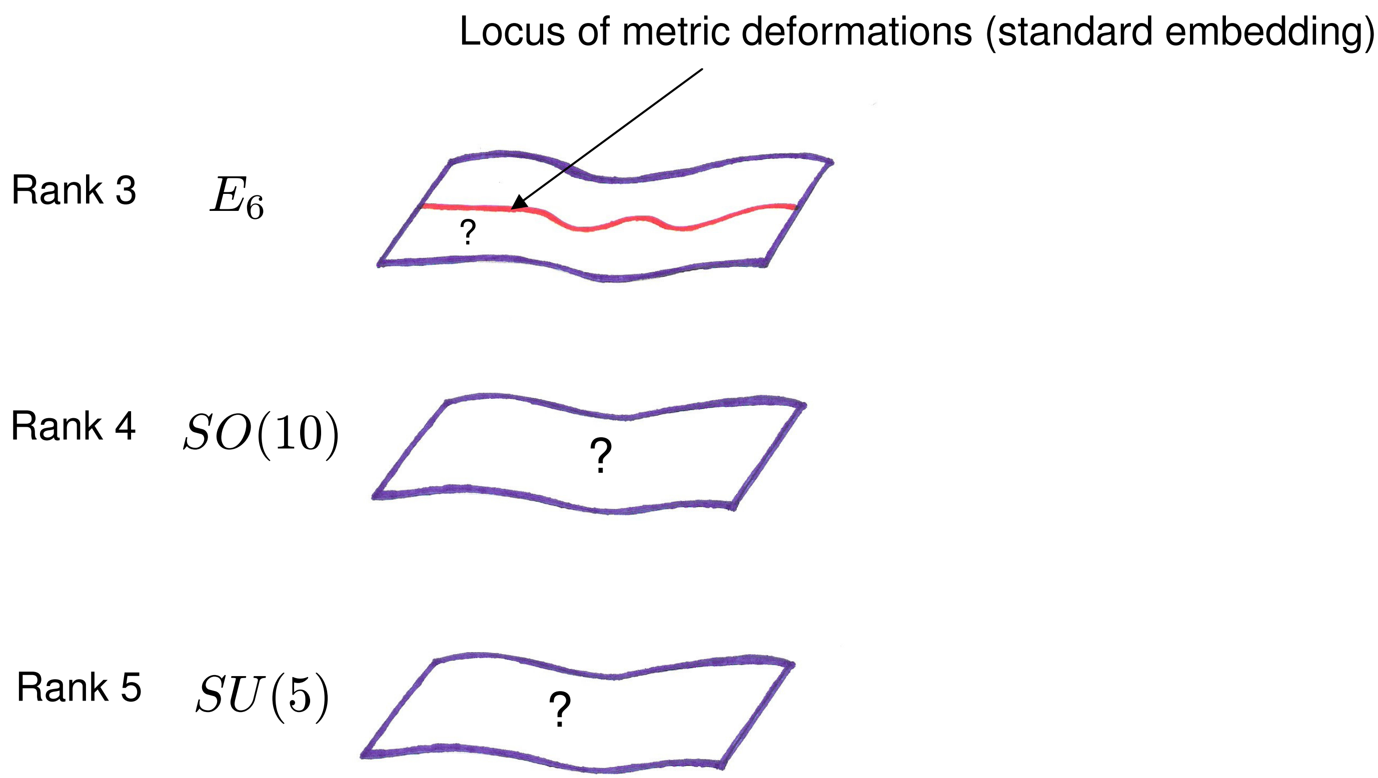
- Compactifications have parameters. Parameter spaces have their own interesting geometry
  - E.g. space of metric deformations (complex structure, Kahler). Well-understood. Special geometry & mirror symmetry.
  - What about bundle deformations?

    …have been largely ignored…

Bundle moduli spaces are large

- Ignoring bundle directions non-generic and artificial.

  CY and bundle deformations mix; not distinguishable



Goals

- Interested two related questions
  - How to construct d=4 EFT?

    Important for phenomenology. After 27 years of heterotic, not been worked out!
  - What is the parameter space geometry (metric, singularities...)? Important for understanding structures of vector bundles

- Rich interplay:
  - d=4 EFT gives induced metric on parameter space.
  - parameter space dictates properties of the family of EFTs

Warm-up: history revisited

- "Simple" solution is the standard embedding: $\omega_\mu = A_\mu$
- E_6 gauge theory with N=1 SUSY
- Parameter space: complexified metric deformations
  Solutions parameterised as

  $$\delta G_{\bar\mu\bar\nu} = \sum_{I=1}^{h^{2,1}} Z^I \chi_{I\,\bar\mu\bar\nu}, \quad \chi_I \in H^1(M,T)$$

  $$\delta B_{\mu\bar\nu} + i\delta G_{\mu\bar\nu} = \sum_{r=1}^{h^{1,1}} T^r e_{r\,\mu\bar\nu}, \quad e_r \in H^1(M,T^*)$$

- Moduli space $\mathcal{M} \leftrightarrow H^1(M,T) \oplus H^1(M,T^*)$
- Dimensional reduction determines natural metric on $\mathcal{M}$

Dimensional reduction gives nice structures

- Parameterisation determines KK ansatz

$$\delta G_{\bar\mu\bar\nu}(x,y) = \sum_{I=1}^{h^{2,1}} Z^I(x)\chi_{I\,\bar\mu\bar\nu}(y)$$

$$\delta B_{\mu\nu}(x,y) + i\delta G_{\mu\bar\nu}(x,y) = \sum_{r=1}^{h^{1,1}} T^r(x) e_{r\,\mu\bar\nu}(y)$$

- Integrate over CY manifold

$$\begin{aligned}\mathcal{L} &= \int_M d^6 y G^{1/2}(R - \frac{1}{2}|H|^2) \\ &= 2g_{r\bar s}\partial_i T^r(x)\partial^i T^{\bar s}(x) + 2g_{I\bar J}\partial_i Z^I(x)\partial^i Z^{\bar J}(x) + \ldots\end{aligned}$$

$$g_{r\bar s} = \frac{1}{4V_6}\int e_r \wedge \star e_s, \qquad g_{I\bar J} = \frac{\int \chi_I \wedge \bar\chi_{\bar J}}{\int \Omega\wedge\bar\Omega}$$

- Kahler: $K_{c.s.} = \log i \int \Omega \wedge \bar\Omega, \quad K_{kah} = \log \frac{4}{3}\int \omega^3$
- Special geometry & mirror symmetry: EFT completely fixed two holomorphic functions (prepotentials)

Where is bundle?

- Study local parameter space about standard embedding. Nice situation, classical moduli space

$$H^1(M,T^*) \oplus H^1(M,T) \oplus H^1(M,\mathrm{End}T)$$

- What is known? $\mathcal{L} = \int d^4\theta K(\Phi,\bar{\Phi}) + \int d^2\theta W(\Phi) + \mathrm{h.c.}$

??

Calculated via e.g. GLSM

- Goal: dimensional reduction to fix D-terms at classical level. Allow $G \longrightarrow G+\delta G, \quad A \longrightarrow A+\delta A$ preserving EOM.

Plan of attack

1. Fix CY, construct metric on bundle deformations
2. Allow CY via complex structure
3. Allow CY via complex structure & Kahler

Fixed CY

- Write $A = \mathcal{A} - \mathcal{A}^\dagger$, and study $\mathcal{A} \to \mathcal{A} + a, \quad \mathcal{A}^\dagger \to \mathcal{A}^\dagger + a^\dagger$
- Need a convenient/natural parameterisation of $a, a^\dagger$ [cf. Z^I, T^r for δG .]
- Introduce parameters w^c so that $\mathcal{A} = \mathcal{A}(x; w)$
- Relation between w^c and $a, a^\dagger$? First guess:

$$\mathcal{A} \to \mathcal{A} + \delta w^c \partial_c \mathcal{A} + \ldots, \quad \text{so} \quad \partial_c A \quad \text{are a basis}$$

Two problems:

1. Not closed: $\partial_c \mathcal{A} \notin H^1(M, \mathrm{End} T)$
2. Not consistent with background gauge transformations:

$$\mathcal{A} \to \Phi \mathcal{A} \Phi^{-1} - \overline{\partial} \Phi \Phi^{-1}, \quad a \to \Phi a \Phi^{-1}$$

(instead $\partial_c \mathcal{A} \to \Phi \Big(\partial_c \mathcal{A} - \overline{\partial}_{\mathcal{A}} [\Phi^{-1} \partial_c \Phi] \Big) \Phi^{-1}$)

Kill two birds with one connection

- Introduce connection Λ_c on moduli space. Transforms:

$$\Lambda_c \to \Phi\Lambda_c\Phi^{-1} - \partial_c\Phi\Phi^{-1}$$

- Can then define covariant derivative $\mathcal{D}_c\mathcal{A} = \partial_c\mathcal{A} - \overline{\partial}_{\mathcal{A}}\Lambda_c$
  - Under gauge transformations: $\mathcal{D}_c\mathcal{A} \to \Phi\mathcal{D}_c\mathcal{A}\Phi^{-1}$
  - Can check that $\overline{\partial}_{\mathcal{A}}(\mathcal{D}_c\mathcal{A}) = 0$
- Can introduce complex structure $w^\alpha, w^{\bar{\alpha}}$ so that $\mathcal{D}_{\bar{\alpha}}\mathcal{A} = 0$
- Natural to expand $a = \delta w^\alpha \mathcal{D}_\alpha \mathcal{A} \quad a^\dagger = \delta w^{\bar{\alpha}} \mathcal{D}_{\bar{\alpha}} \mathcal{A}^\dagger$

Kobayashi Metric

- Now repeat dimensional reduction exercise (fun fun fun!)

$$\mathcal{L} \;=\; \frac{\alpha'}{4}\int_M d^6y e^{-2\phi}G^{1/2}\mathrm{Tr}F_{MN}F^{MN} = 2g_{\alpha\bar\beta}\partial_i W^\alpha(x)\partial^i W^{\bar\beta}(x) + \ldots$$

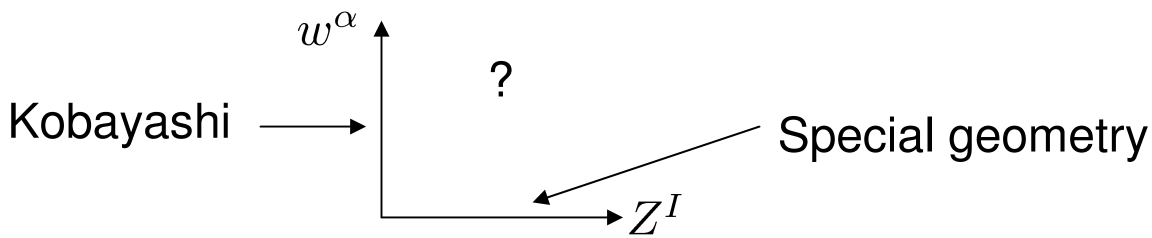
where $g_{\alpha\bar\beta} = \frac{i\alpha'}{8V_6}\int_M \omega^2 \mathrm{Tr}\mathcal{D}_\alpha\mathcal{A}\mathcal{D}_{\bar\beta}\mathcal{A}^\dagger$

- Manifestly hermitian. Can show its Kahler: $\partial_\gamma g_{\alpha\bar\beta} = \partial_\alpha g_{\gamma\bar\beta}$
- Agrees with Weil-Peterson metric on $H^1(M, \mathrm{End}T)$ [Kobayashi]
- Kahler potential: $K_{\mathrm{bun}} = \frac{i\alpha'}{8V_6}\int \omega^2 \mathrm{Tr}\mathcal{A}\mathcal{A}^\dagger$

Curiously, defined in a gauge: $g^{\mu\bar\nu}\partial_\mu\mathcal{A}_{\bar\nu} = 0$

Beyond the wheel: complex structure

- Now allow CY to vary. $G = G(Z^I), \quad \mathcal{A} = \mathcal{A}(Z^I, w^\alpha)$



- Need to find nice parameterisation for KK ansatz.
  Introduce: $\mathcal{D}_I \mathcal{A} = \partial_I \mathcal{A} + \chi_I^\rho \mathcal{A}_\rho^\dagger + \overline{\partial}_\mathcal{A} \lambda_I^\dagger$

  Implicit c.s. dependence of $\mathcal{A}$

- Parameterise fluctuations:

$$a = \delta w^\alpha \mathcal{D}_\alpha \mathcal{A} + \delta Z^I \mathcal{D}_I \mathcal{A}$$

$$a^\dagger = \delta w^{\bar{\alpha}} \mathcal{D}_\alpha \mathcal{A}^\dagger + \delta Z^{\bar{I}} \mathcal{D}_{\bar{I}} \mathcal{A}^\dagger$$

$$\delta G = \delta Z^I \chi_I$$

Complex-Structure---Bundle Metric

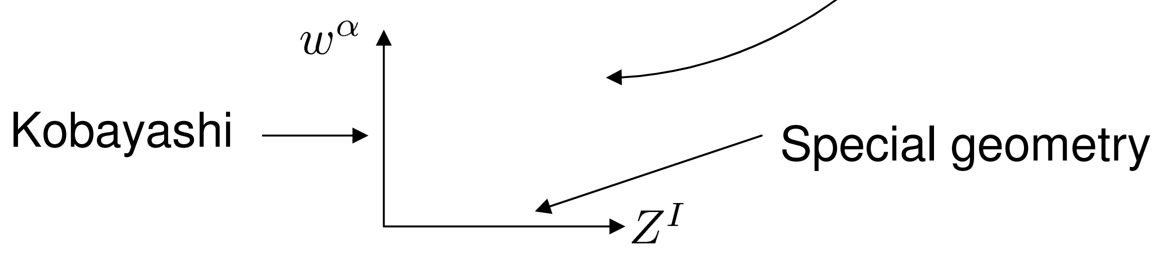
- Repeat the exercise. Find a hermitian moduli space metric

$$ds^2 = g_{\alpha\bar\beta}(dw^\alpha - C^\alpha_I dZ^I)(dw^{\bar\beta} - C^{\bar\beta}_{\bar J} dZ^{\bar J}) + g^{s.g.}_{I\bar J} dZ^I dZ^{\bar J}$$

where $g^{s.g.}_{I\bar J}$ is the special geometry, while

$$C^\alpha_I = g^{\alpha\bar\beta} \frac{i\alpha'}{8V_6} \int_M \omega^2 \mathrm{Tr} \mathcal{D}_I \mathcal{A} \mathcal{D}_{\bar\beta} \mathcal{A}^\dagger$$

- Metric is Kahler
- Fibration structure



Kahler parameters

- Allow fields to depend on Kahler parameters. Distinguished from c.s. case $g_{\alpha\bar{\beta}} = \frac{i\alpha'}{8V_6}\int_M \omega^2 \mathrm{Tr}\mathcal{D}_\alpha\mathcal{A}\mathcal{D}_{\bar{\beta}}\mathcal{A}^\dagger$

  Depends explicitly on Kahler class

- If naively follow the well-trodden path, find a non-Kahler metric! Resolution appears to be a modification:

$$\mathcal{D}_r\mathcal{A} = \partial_r\mathcal{A} + \partial_r K_{\mathrm{Kah}} + \overline{\partial}_{\mathcal{A}}\lambda_r^\dagger$$

  Implies a "gauge symmetry" over Kahler moduli space.

Final Metric!

- End result is

$$
\begin{aligned}
ds^2 \quad = \quad & g_{\alpha\bar{\beta}}(dw^\alpha - C^\alpha_I dZ^I - D^\alpha_s dt^s)(dw^{\bar{\beta}} - C^{\bar{\beta}}_{\bar{J}} dZ^{\bar{J}} - D^{\bar{\beta}}_{\bar{r}} dt^{\bar{r}}) + \\
& 2 g_{r\bar{\beta}} dt^r dw^{\bar{\beta}} + 2 g_{\alpha\bar{s}} dw^\alpha dt^{\bar{s}} + \text{special geometry}
\end{aligned}
$$

- Metric no longer an obvious fibration thanks to CS(A) twisting of H. But term is necessary for Kahlerity.
- Kahler potential: $K = \log \frac{4}{3} \int (\omega + \frac{i\alpha'}{4} \mathrm{Tr} \mathcal{A}\mathcal{A}^\dagger)^3 + \log i \int \Omega \wedge \bar{\Omega}$

First fundamental form of principal bundle restricted to the CY

Categorification

Summary

- Taken some first steps towards identifying a local moduli space metric.
- Identified a nice parameterisation via gauge invariance.
- Many Questions remain:
  - Matter field metric?
  - Quantum corrections?
  - Mirror symmetry, and remnants of special geometry?