Generalized type IIA orientifold and M-theory compactifications

Hagen Triendl

IPhT/CEA Saclay

String Math 2012
18th July

based on work in progress with M. Graña

Generalized Geometry

- Supergravity (and string theory) has more symmetries than just diffeomorphisms
- E.g. symmetry group of metric and B-field for compactification on T^n :

$$\mathbf{SO(6,6)}$$

- Locally, all backgrounds have this symmetry group
- $\mathbf{SO(6,6)}$ acts covariantly on $TM \oplus T^*M$
- Fields combine into metric on bundle:

$$\begin{pmatrix} g_{mn} - B_{mp}B_{nq}g^{pq} & -B_{mp}g^{pn} \\ -B_{np}g^{pm} & g^{mn} \end{pmatrix}$$

- Formalism enables to deal with general flux compactifications

G-structure backgrounds

- Backgrounds with (spontaneously broken) supersymmetry must admit *nowhere-vanishing* spinors $\boldsymbol{\eta^i}$:

- This induces a certain *G-structure* on the background:

$$SU(3) \subset Gl(6) \qquad (\eta^1 = \eta^2)$$

$$SU(3) \times SU(3) \subset SO(6,6)$$

- Such a G-structure is characterized by *nowhere-vanishing* objects whose common stabilizer is G.

For $SU(3)$: $\boldsymbol{J}$ and $\boldsymbol{\Omega}$

- Supersymmetry imposes 1st order differential conditions on these objects – can be recast in superpotential

The NS5 and generalized geometry

- The NS5-brane gives contribution to Bianchi identity of $\boldsymbol{H_3}$:

$$\mathbf{d}\boldsymbol{H_3} = \boldsymbol{t_{NS5}\delta^4(x)}$$

- Therefore, the B-field is not well-defined any more at the position of the NS5-brane.

- The "twisted" differential $\mathbf{d}_{\boldsymbol{H}} = \mathbf{d} - \boldsymbol{H_3}\wedge$ does not square to zero any more:

$$\mathbf{d}_{\boldsymbol{H}}{}^{\mathbf{2}} = \boldsymbol{t_{NS5}\delta^4(x)}$$

- If we covariantize more symmetries, we have to include more fields. The problem of describing sources becomes worse!

Duality between type IIA and M-theory

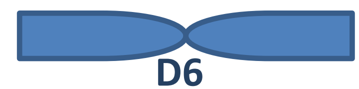
Type IIA on M ⟺ duality ⟺ M-theory on M x S^1

(g_{MN}, C_1, ϕ) $\tilde{g}_{\tilde{M}\tilde{N}}$

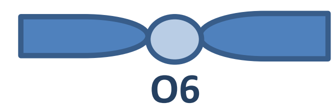
(B_2, C_3) $\tilde{C}_3$

D6-brane ⟷ Taub-NUT

Lu, Duff '93; Townsend '95; Sen '97; …



O6-plane ⟷ Atiyah-Hitchin



O6-orientifold ⟺ duality ⟺ Orbifold (M x S^1)/$\mathbb{Z}_2$

SO(4) gauge group ⟷ 2x **A1** singularity

Kodaira classification

D6-brane d.o.f. blow-up modes

Topics

How to covariantize all symmetries of supergravity

Describe G-structures

Type IIA – M-theory duality in generalized geometry

Is there a way to describe sources in generalized geometry?

Symmetries in Supergravity

	Type IIA	M-theory	
geometric	$\mathbf{Gl(6,\mathbb{R})}$	$\mathbf{Gl(7,\mathbb{R})}$	
	↓	↓	
(NS-NS)	$\mathbf{SO(6,6)\times Sl(2,\mathbb{R})}$	$\mathbf{Sl(8,\mathbb{R})}$	(metric and $\tilde{C}_6$)
	↘	↙	
full supergravity	$\mathbf{E_{7(7)}}$		
	↓		
string theory	$\mathbf{E_{7(7)}(\mathbb{Z})}$		Hull, Townsend '95

Generalized Geometry in type IIA

- Covariant formulation:

symmetry group	bundle	
$\mathbf{Gl}(6,\mathbb{R})$	TM	
$\mathbf{SO}(6,6)$	$TM \oplus T^*M$	Hitchin '01; Gualtieri '04;…
$\mathbf{E}_{7(7)}$	$TM \oplus T^*M \oplus \Lambda^5 TM \oplus \Lambda^5 T^*M \oplus \Lambda^{\text{even}} T^*M$	Hull'08; Pacheco, Waldram '09; Graña, Louis, Sim, Waldram '09

- Embed spinors η^i into (pure) spinors of $\mathbf{SO}(6,6)$:

$$\eta^i \longrightarrow \begin{array}{l} \Phi^+ = e^{-B}\eta^1 \otimes \bar{\eta}^2 \\ \Phi^- = e^{-B}\eta^1 \otimes \left(\bar{\eta}^2\right)^c \end{array}$$

($\Phi^\pm$ in $\Lambda^{\text{even/odd}} T^*M$)

- then embed into $\mathbf{E}_{7(7)}$ language

SU(6)-structures in IIA

Graña, Louis, Sim, Waldram '09

- $\mathbf{E}_{7(7)}$ representations in terms of $\mathbf{SO(6,6)} \times \mathbf{Sl(2,\mathbb{R})}$:

$$\mathbf{56} = (\mathbf{12},\mathbf{2}) \oplus (\mathbf{32}^+,\mathbf{1})$$

$$\mathbf{133} = (\mathbf{66},\mathbf{1}) \oplus (\mathbf{1},\mathbf{3}) \oplus (\mathbf{32}^-,\mathbf{2})$$

- Embed pure spinors $\boldsymbol{\Phi}^{\pm}$ into $\mathbf{E}_{7(7)}$ representations:

$$\boldsymbol{L} = \boldsymbol{e}^{\boldsymbol{C}}(\mathbf{0},\boldsymbol{\Phi}^+)$$ ⇨ Kähler space

complex str.: $\boldsymbol{J} = \boldsymbol{L} \times \boldsymbol{L}$

$$\boldsymbol{e}^{\boldsymbol{K}} = \boldsymbol{q}(\boldsymbol{L}) = -\mathbf{tr}(\boldsymbol{J}^2)$$

$$\boldsymbol{H}_+ = \boldsymbol{e}^{\boldsymbol{C}}(\mathbf{0},\mathbf{0},\boldsymbol{\tau}^{\boldsymbol{i}}\boldsymbol{\Phi}^-)$$

$$\boldsymbol{H}_- = \overline{\boldsymbol{H}_+}$$

$$\boldsymbol{H}_3 = [\boldsymbol{H}_+,\boldsymbol{H}_-]$$

} c-map → Quaternion-Kähler space

$\boldsymbol{H}_{\boldsymbol{a}}$ form $\mathbf{SU(2)}$ R-symmetry

- Prepotentials $\boldsymbol{P}^{\boldsymbol{a}}$ give $\boldsymbol{N} = \mathbf{2}$ couplings: $\boldsymbol{P}^{\boldsymbol{a}} = \langle \boldsymbol{D}\boldsymbol{H}_{\boldsymbol{a}}, \boldsymbol{L}\rangle$

Generalized Geometry in M-theory

Hull'08; Pacheco, Waldram '09;

- Use covariant formulation:

symmetry group	bundle	fields
$\mathbf{Gl}(7,\mathbb{R})$	TM	g_{mn}
$\mathbf{Sl}(8,\mathbb{R})$	$T_8M = \left(TM \oplus \Lambda^7 TM\right)_0$	g_{mn}, C_6
$\mathbf{E}_{7(7)}$	$\Lambda^2 T_8 M \oplus \Lambda^2 {T_8}^* M$	g_{mn}, C_6, C_3

- For reduction to type IIA:

$$TM = T_6 M \oplus T_v M$$

- We call the one-forms on $T_v M$ and $\Lambda^7 TM$ by v and ρ resp.

N=2 backgrounds in M-theory

Graña, HT '12

- We start with two **orthonormal** spinors η^i (since $N = 2$)
- Embed η^i as bilinears into $\mathrm{E}_{7(7)} \to \mathrm{SU}(8)$ representations:

$$\mathbf{56} = \mathbf{28} \oplus \overline{\mathbf{28}} \qquad \mathbf{133} = \mathbf{63} \oplus \mathbf{70}$$

$$L = \epsilon_{ij} e^{C_3 + C_6} (\eta^i \otimes \eta^j, 0)$$
$$\bar{L} = \epsilon_{ij} e^{C_3 + C_6} (0, \bar{\eta}^i \otimes \bar{\eta}^j)$$

$$H_a = (\sigma_a)_{ij} e^{C_3 + C_6} (\eta^i \otimes \bar{\eta}^j, 0)$$

- These objects have all the properties of their IIA counterparts

SU(6) structure

Emergence of an 8d space?

Graña, HT '12

- Both $\boldsymbol{L}$ and $\boldsymbol{H_a}$ have a nice interpretation in terms of $\boldsymbol{T_8 M}$:

$$\boldsymbol{L} = -e^{C_4}\left(\ast_8 \left(J_8 \wedge J_8 \wedge J_8\right), \mathrm{i} J_8\right)$$

$$\boldsymbol{H_+} = e^{C_4}(0, \Omega_4) \qquad \boldsymbol{H_-} = e^{C_4}(0, \bar{\Omega}_4) \quad \boldsymbol{H_3} = e^{C_4}(I_8, 0)$$

- This describes an $\mathbf{SU(4)}$ structure on $\boldsymbol{T_8 M}$:

$$\Omega_4 = (\rho + (C_6 \wedge v) + \mathrm{i} v) \wedge \Omega$$

$$J_8 = J + v \wedge \rho \qquad\qquad C_4 = \rho \wedge C_3$$

- This reminds of F-theory, but form fields (and SUSY) are different

SU(7) structures in M-theory

Pacheco, Waldram '09; Graña, HT '12

- Now only one spinor η ➡ no nice embedding in **56** or **133**
- However, we have for $\mathbf{E}_{7(7)} \to \mathbf{SU(8)}$:

$$\mathbf{912} = \mathbf{36} \oplus \mathbf{420} \oplus \overline{\mathbf{36}} \oplus \overline{\mathbf{420}}$$

$$\Phi = e^{C_3 + C_6}(\eta \otimes \eta, 0, 0, 0)$$

- Again, nice form on T_8M:

$$\Phi = e^{C_4}(g_8{}^{-1}, g_8 \cdot (\epsilon^{-1} \cdot \phi_4), \mathrm{i} g_8, \mathrm{i} g_8{}^{-1} \cdot \phi_4)$$

where g_8 is metric on T_8M and

$$\phi_4 = \rho \wedge \alpha + *_7\, \alpha, \quad C_4 = \rho \wedge C_3$$

N=1 backgrounds in M-theory

House, Micu'04;
Pacheco, Waldram '09;
Graña, HT '12

- In almost complex structure, g_8 drops out:

$$J_{\Phi} = \Phi \times \Phi = e^{C_4}(0, \phi_4)$$

- Kähler potential is given by quartic invariant:

$$e^K = \int q(\mathrm{Re}\Phi) = \int -\mathrm{tr}\left(J_{\Phi}{}^2\right) = \int \phi_4 \wedge \phi_4$$

- Superpotential is determined by EV equation: $(D\Phi)\cdot\Phi = W\Phi$

$$\Rightarrow W = \int \iota_\rho \phi_4 \wedge \mathrm{d}\phi_4 - 2\mathrm{i}\phi_4 \wedge \iota_\rho \mathrm{d}C_4 + C_4 \wedge \iota_\rho \mathrm{d}C_4$$

Generalized Orientifolding

Graña, HT '12

- Orientifold action on $\mathbf{E_{7(7)}}$ representations:

$$\sigma(L) = -\lambda(L), \quad \sigma(H_1) = \lambda(H_1), \quad \sigma(H_{2/3}) = -\lambda(H_{2/3}),$$

- When going from IIA to M-theory, orientifolding becomes orbifolding:

$$\sigma_{\text{M-theory}} = \sigma_{\text{IIA}} \circ \lambda,$$

- The objects L and $H_{2/3}$ do not survive orbifolding, but

$$\mathbf{912} \subset \mathbf{133} \otimes \mathbf{56}$$

$\Phi = (H_2 + \mathrm{i} H_3) \times L$ and $J_\Phi = H_1 + J_L$ do!

Generalized Orientifolding

Graña, HT '12

- At fixed points of σ , the bundle degenerates:

$$\mathbf{E}_{7(7)} \text{ is broken to } \mathbf{SO(6,6)} \times \mathbf{Sl(2,\mathbb{R})}$$

- Blow-ups restore $\mathbf{E}_{7(7)}$ symmetry, but

SU(6) structure → SU(7) structure

- The $N = 2$ quantities descend to the $N = 1$ quantities in the usual way, in particular:

$$W = e^{-K_\Phi}(P_2 + \mathrm{i}P_3)$$

Concluding Remarks

- $\mathbf{E}_{7(7)}$-generalized geometry gives a elegant and powerful formalism to relate type IIA and M-theory backgrounds

- It is not clear if the eight-dimensional bundle hints to some eight-dimensional space like in F-theory

- New Orientifold backgrounds from $\mathbf{E}_{7(7)}$ transformations (like NS5-O-planes)

- Sources get regularized by introduction of extra coordinates (e.g. D6-branes become regular sources in M-theory)

Hint to double field theory?