Non-geometric fluxes versus (non)-geometry

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arXiv:1106.4015 by D. A., M. Larfors, D. Lüst, P. Patalong
arXiv:1202.3060 by D. A., O. Hohm, M. Larfors, D. Lüst, P. Patalong
arXiv:1204.1979 by D. A., O. Hohm, M. Larfors, D. Lüst, P. Patalong
and work in progress...

18/07/2012, String-Math 2012,
Bonn, Germany

Introduction

Non-geometry in 10d and 4d SUGRA

Restrict to NSNS sector: $g_{mn},\ b_{mn},\ \phi$.

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  $\hookrightarrow$ use stringy symmetries for gluing

hep-th/0208174 by S. Hellerman, J. McGreevy, B. Williams

hep-th/0210209 by A. Dabholkar, C. Hull

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  ($\Leftrightarrow$ specific gaugings in gauged SUGRA).

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  Terms ✓ for pheno. : stab. of moduli, de Sitter sol. ...

  hep-th/0607015 by J. Shelton, W. Taylor, B. Wecht, hep-th/0701173 by A. Micu, E. Palti, G. Tasinato, arXiv:0911.2876 by B. de Carlos, A. Guarino, J. M. Moreno

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    $\hookrightarrow$ manifestly covariant action w.r.t diffeomorphisms
    R-flux: tensor, Q-flux: connection $\Rightarrow$ geom. role

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  - relations between non-geometry, the new fields, and non-commutative geometry

Field redefinition

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Motivations from Generalized Complex Geometry/SUGRA
Arguments in GCG: β related to non-geometry / to ${Q_k}^{mn}$, R^{kmn}

hep-th/0609084, arXiv:0708.2392 by P. Grange, S. Schäfer-Nameki

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$$\mathcal{H} = \begin{pmatrix} g - bg^{-1}b & bg^{-1} \\ -g^{-1}b & g^{-1} \end{pmatrix} = \begin{pmatrix} \tilde{g} & -\tilde{g}\beta \\ \beta\tilde{g} & \tilde{g}^{-1} - \beta\tilde{g}\beta \end{pmatrix}, \ \tilde{g}: \text{ new metric}$$

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$$\Leftrightarrow \quad \begin{array}{l} g = (\tilde{g}^{-1} + \beta)^{-1}\tilde{g}^{-1}(\tilde{g}^{-1} - \beta)^{-1} \\ b = -(\tilde{g}^{-1} + \beta)^{-1}\beta(\tilde{g}^{-1} - \beta)^{-1} \end{array} \quad \Leftrightarrow \quad \begin{array}{c} (g + b) = (\tilde{g}^{-1} + \beta)^{-1} \\ \text{(useful for DFT)} \end{array}$$

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$$e^{-2\tilde{\phi}}\sqrt{|\tilde{g}|} = e^{-2\phi}\sqrt{|g|}$$

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Apply it on NSNS Lagrangian?
β could be related to non-geo. fluxes $\Rightarrow$ would they appear?

Rewriting of the NSNS Lagrangian

$$\mathcal{L} = e^{-2\phi}\sqrt{|g|}\left(\mathcal{R}(g) + 4(\partial\phi)^2 - \frac{1}{2.3!}H_{kmn}H_{pqr}\ g^{kp}g^{mq}g^{nr}\right)$$

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(assumption: $\beta^{km}\partial_m \cdot = 0$)

$$\begin{aligned}
\mathcal{R}(g) = \mathcal{R}(\tilde{g}) &- \partial_k\tilde{g}_{su}\partial_m\tilde{g}_{pq}\left(2\tilde{g}^{km}\tilde{g}^{uq}\tilde{g}^{ps} + 2\tilde{g}^{pq}\tilde{g}^{ks}\tilde{g}^{mu} + \frac{1}{2}\tilde{g}^{uq}\tilde{g}^{sm}\tilde{g}^{kp}\right)\\
&- \tilde{g}_{pq}\partial_k\beta^{pk}\partial_m\beta^{qm} - \frac{1}{2}\tilde{g}_{pq}\partial_k\beta^{qm}\partial_m\beta^{pk}\\
&+ 2\tilde{g}^{km}\tilde{g}^{pq}\partial_k\partial_m\tilde{g}_{pq} + 2\tilde{g}^{km}(G^{-1})_{pq}\partial_k\partial_m G^{qp}\\
&+ \partial_m G^{vl}\Big(-2\tilde{g}^{mr}\tilde{g}^{ks}(G^{-1})_{lv}\partial_k\tilde{g}_{rs} - \tilde{g}^{rs}\tilde{g}^{km}(G^{-1})_{lv}\partial_k\tilde{g}_{rs}\\
&\quad + \tilde{g}^{ms}\tilde{g}^{ru}(G^{-1})_{lu}\partial_v\tilde{g}_{rs} - \tilde{g}^{km}\tilde{g}^{rs}(G^{-1})_{ls}\partial_k\tilde{g}_{vr}\Big)\\
&+ \partial_m G^{vl}\Big((G^{-1})_{lq}\partial_v G^{qm} + \frac{1}{2}g_{lq}\partial_v G^{mq}\Big)\\
&- \partial_m G^{vl}\partial_k G^{ps}\frac{1}{2}\tilde{g}^{km}\Big(2(G^{-1})_{lv}(G^{-1})_{sp} + 5(G^{-1})_{sv}(G^{-1})_{lp} + g_{sl}\tilde{g}_{pv}\Big)\\
&\text{where } G = \tilde{g}^{-1} + \beta\ .
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$$= e^{-2\tilde{\phi}}\sqrt{|\tilde{g}|}\left(\mathcal{R}(\tilde{g}) + 4(\partial\tilde{\phi})^2 - \frac{1}{2}|Q|^2\right) + \partial(\dots) = \tilde{\mathcal{L}} + \partial(\dots)$$

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where ${Q_k}^{mn} = \partial_k\beta^{mn}$, $|Q|^2 = \frac{1}{2!}{Q_k}^{mn}{Q_p}^{qr}\ \tilde{g}^{kp}\tilde{g}_{mq}\tilde{g}_{nr}$

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$\Rightarrow$ also get $R^{mnp} = 3\ \beta^{k[m}\partial_k\beta^{np]}$, $|R|^2 = \frac{1}{3!}R^{kmn}R^{pqr}\ \tilde{g}_{kp}\tilde{g}_{mq}\tilde{g}_{nr}$

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$\hookrightarrow$ action, integration $\Rightarrow$ global aspects

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In some examples, $\tilde{\mathcal{L}}(\tilde{g}, \beta, \tilde{\phi})$ ✓

We propose to discard $\partial(\dots)$, use $\tilde{\mathcal{L}}$ as the good low-energy effective description of string theory

Global aspects

We have shown

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Relation between 10d/4d non-geometry:
for a 10d non-geometric configuration,
field redefinition + dimensional reduction
$\Rightarrow$ generates 4d non-geometric terms

More geometrical structure

Double field theory

arXiv:0904.4664, arXiv:0908.1792 by C. Hull, B. Zwiebach

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Q-, R-fluxes help to make diffeomorphism cov. manifest in DFT
+ they get a geometrical role: R is a tensor
Q is not a tensor, rather a connection

Relations to non-commutative geometry

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- Open string: T-dual of D-brane with a constant b-field B leads to $[x^i(\tau), x^j(\tau)] = \mathrm{i}\theta^{ij}$, θ non-com. parameter

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hep-th/9908142 by N. Seiberg, E. Witten

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arXiv:1010.1361 by D. Lüst

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- Non-associativity: R-flux is the parameter... $[[X^i(\tau,\sigma), X^j(\tau,\sigma)], X^k(\tau,\sigma)] + \text{perm.} \sim R^{ijk}$

(see talk of A. Deser)

R. Blumenhagen, et al.

Conclusion and outlook

- GCG $\Rightarrow$ field redefinition $(g, b, \phi) \leftrightarrow (\tilde{g}, \beta, \tilde{\phi})$
  Rewriting NSNS Lag.: $\mathcal{L}(g, b, \phi) = \tilde{\mathcal{L}}(\tilde{g}, \beta, \tilde{\phi}) + \partial(\dots)$
  10d ${Q_k}^{mn} = \partial_k \beta^{mn}$ (for $\beta^{km}\partial_m \cdot = 0$), $R^{mnp} = 3\ \beta^{k[m}\partial_k \beta^{np]}$
- 10d NSNS non-geometry $\Rightarrow \tilde{\mathcal{L}}(\tilde{g}, \beta, \tilde{\phi})$ has no global issue...
  $\hookrightarrow$ dim. reduction of $\tilde{\mathcal{L}} \Rightarrow$ 4d non-geometric potential ✓
  Relation between 4d/10d non-geometry
- Manifestly covariant DFT action w.r.t. diffeomorphisms
  Geometrical role of non-geometric fluxes:
  R: tensor, Q: connection
- Non-commutativity, non-geometry and field redefinition
  Study relations, get more closed string examples...
- Extend to RR sector (S-duality)
  and D-brane/O-plane sources (new objects?)
  $\hookrightarrow$ new interesting backgrounds, 10d de Sitter solutions...

The dimensional reduction

The 4d scalar potential

Split 10d $\Rightarrow$ 4d max. sym. space-time $\times$ 6d compact $\mathcal{M}$
Compactification ansatz: $\mathrm{d}s_{10}^2 = \mathrm{d}s_4^2 + \mathrm{d}s_6^2$ (no warp factor), b, β purely internal

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Only two 4d scalar fields: volume ρ and dilaton σ:

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$$S_E = M_4^2 \int \mathrm{d}^4x\ \sqrt{|g^E|} \left(\mathcal{R}_4^E + \mathrm{kin} - \frac{1}{M_4^2} V(\rho,\sigma) \right)$$

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arXiv:0711.2512 by M. P. Hertzberg, S. Kachru, W. Taylor, M. Tegmark

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With $\tilde{\mathcal{L}}$ instead of $\mathcal{L}$, we get the ✓ 4d potential

We get $V(\rho,\sigma) = \sigma^{-2}\left(\rho^{-1}\ V^0_{\mathcal{R}} + \rho\ V^0_Q + \rho^3\ V^0_R\right)$ where

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$\tilde{\mathcal{L}}$ and 10d Q, R give the ✓ 4d potential (give a 10d origin)

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4d Q is not clearly identified...
$\hookrightarrow$ DFT brings an interpretation for the Q-terms, the role of Q