Heterotic flux vacua and their IIA duals

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a résumé of ArXiv:1206.1417, with R. Minasian and S. Theisen

Motivation

Heteroptic string offers a rich interplay of geometry and physics

. . . and thus lots of questions!

- which geometries lead to perturbative string vacua (i.e. CFTs)?
- which of those exist non-perturbatively?
- what is the structure of moduli spaces and possible transitions?

Motivation

N=1, $\mathbb{R}^{1,3}$ vacua have been a major focus

- phenomenology (heterotic and F)
- stringy geometry (worldsheet instantons, quantum sheaf cohomology)
- flux and non-geometric vacua

this talk : N=2 $\mathbb{R}^{1,3}$ perturbative heterotic vacua and their duals

- quantum corrections under better control, yet far from trivial
- an elegant targetspace geometry
- more general perspective on heterotic / IIA duality

Outline

- N=2 heterotic compactification: CFT and geometry
- Flux and its consequences
- A $d = 8$ perspective
- IIA / heterotic duality
- Bright future

Worldsheet vs spacetime supersymmetry

Classic result 1 [Banks, Dixon, Friedan & Martinec,'88]

critical perturbative heterotic $\mathbb{R}^{1,3}$ vacuum has N=1 SUSY

$$\Longleftrightarrow$$

(0,1) internal SCFT $\to$ (0,2) SCFT, $\overline{q} \in \mathbb{Z}$

Classic result 2 [Hull&Witten,'85; Sen,'86; Strominger, '86; Hull&Townsend, '86]

if geometric realization by (0,1) NLSM with target X then:

X complex 3-fold,	$K_X = \mathcal{O}_X$;
$d(J \wedge J e^{-2\varphi}) = 0$,	$\mathcal{H} = i(\bar{\partial} - \partial)J$;
$E \to X$, HYM $\mathcal{A}$,	$d\mathcal{H} = \frac{\alpha'}{4}(\mathrm{tr}\{\mathcal{R}_+^2\} - \mathrm{tr}\{\mathcal{F}^2\})$;
$\mathfrak{g}_E \subset \mathfrak{so}(32)$ or $\mathfrak{so}(16)^2$,	$w_1(E) = w_2(E) = 0$.

Worldsheet vs spacetime supersymmetry

Classic result 3 [Banks&Dixon, '88; Lauer, Lüst & Theisen, '88]

$$\text{perturbative heterotic } \mathbb{R}^{1,3} \text{ vacuum has N=2 SUSY}$$
$$\Longleftrightarrow$$
$$\{N=2\}_{\overline{c}=9} \text{ SCA} \to \{N=2\}_{\overline{c}=3} + \{N=4\}_{\overline{c}=6}.$$

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Result : if geometric realization by NLSM with target X then:

$X \xrightarrow{\pi} M$ is principal T^2 bundle,	M is K3, CY $\widehat{g}$;
$g = \pi^* e^{2\varphi}\widehat{g} + \mathcal{G}_{IJ}\Theta^I\Theta^J$,	$\Theta^I = d\theta^I + \pi^* A^I$;
$F^I = dA^I$ are ASD,	$\mathcal{H} = \widehat{\mathcal{H}} - \mathcal{G}_{IJ}\Theta^I F^J$;
$\mathcal{A} = \pi^*\widehat{\mathcal{A}} + \mathbf{a}_I\Theta^I$	$\widehat{\mathcal{A}}$ HYM on M,
$\mathbf{a}_I$ are the Wilson lines,	$\mathcal{G}$ is constant T^2 metric.

cf heterotic fluxes[Dasgupta, Rajesh, Sethi '99; Goldstein&Prokushkin '02, Becker et al '06; Fu&Yau '06]

Flux and its consequences

- constraints on K3 geometry (cf Abelian instantons on K3 [Honecker '06])

  K3 M, CY $\widehat{g}$ determined by J, Ω in $H^2(M)$

  $X \to M$ principal T^2 bundle, $F^I = dA^I \in H^2(M, 2\pi\mathbb{Z})$

  F^I ASD $\iff J \wedge F^I = \Omega \wedge F^I = 0 \implies$ restrictions on $\widehat{g}$

  $\delta\widehat{B} \sim \epsilon_I F^I \implies \delta\widehat{B} \propto F^I$ not a deformation

  there is no sensible split in (0,2) SCFT moduli space

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- constraints on T^2

  naively can choose $\mathcal{G}$ and $b = B_{12}$ freely

  flux quantization $\implies b$ and $\mathcal{G}^*_{IJ} \equiv \mathcal{G}_{IJ} - \frac{\alpha'}{4}\,\mathrm{tr}\{\mathbf{a}_I \mathbf{a}_J\}$ quantized.

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We can count first order deformations;

N=2 spacetime SUSY ensures these are integrable

flux compactification $\iff X$ is non-trivial T^2 bundle

The $d = 8$ perspective

A gradual compactification[Kachru&Vafa '95]

1. compactify heterotic string on T^2

   $\Longrightarrow$ $d = 8$ SUGRA + SYM with gauge group G

2. compactify further on a K3 M with $k = 24$ instanton $\subset G$

   $\Longrightarrow$ $d = 4$ N=2 SUGRA + axio-dilaton vector + $H \subset G$ SYM
   + N_H^0-neutral + N_H^+-charged hypermultiplets

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This is a sign of a heterotic flux vacuum!

The $d = 8$ perspective

Classic example [Kachru&Vafa '95]

1. compactify on T^2 with $\tau = \rho$, $G = \mathrm{U}(1) \times \mathrm{SU}(2) \times \mathrm{E}_8 \times \mathrm{E}_8$

2. instanton : $\mathrm{SU}(2)_{k=4} \times \mathrm{SU}(2)_{k=10} \times \mathrm{SU}(2)_{k=10} \subset \mathrm{SU}(2) \times \mathrm{E}_8 \times \mathrm{E}_8$

   gauge group: $H = \mathrm{U}(1)^2 \times \mathrm{E}_7^2$ + graviphoton+axio-dilaton vector

3. on Higgs branch: $H = \mathrm{U}(1)^2$, $N_H^0 = 129$

   $\leftrightarrow$ IIA dual : $Y_{12} \subset \mathbb{P}^4_{11226}$; detailed tests [Kaplunovsky,Louis & Theisen '95]

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an SU(2) WZW model with (0,2)+(0,4) SUSY fibered over M
(cf [Distler & Sharpe, '07; Adams & Guarrera, '09])

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Result: heterotic flux vacua lead to many more examples with small H;
e.g. potential duals of K3-fibered CYs with $h^{1,1} = 2$, $h^{1,2} = 44, 62$
(codim 2 complete intersections [Klemm, Kreuzer, Riegler & Scheidegger '04])

IIA / heteroric flux duality

Classic result 4 [Aspinwall & Louis '95; Morrison & Vafa '96; Aspinwall & Gross '96]

perturbative heterotic compactification	$\leftrightarrow$	IIA on smooth CY 3-fold Y
weak coupling	$\leftrightarrow$	a large radius limit,

$\Longrightarrow$ Y is K3-fibered; Kähler class dual to generic fiber maps to axio-dilaton; moreover, if heterotic theory has a $d = 6$ decompactification limit, then Y has a compatible elliptic fibration with section.

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Proposal : Let Y be a K3-fibered CY 3-fold without compatible elliptic fibration with section; its weakly-coupled heterotic dual, if it exists, will be an N=2 heterotic flux vacuum.

Outlook

- Unlike their $N = 1$ relatives, $N = 2$ perturbative heterotic flux compactifications are quite constrained and controlled.
- Duality offers hope of determining non-perturbative corrections.
- Many tractable examples for study and exploration!
- We should test proposed dual pairs by examining the vector moduli space.
- It would be nice to develop a better picture of heterotic T-duality orbits [Evslin & Minasian '09]; interesting effects on the IIA side?
- What is the F-theory perspective on this class of theories?