Understanding logarithmic CFT

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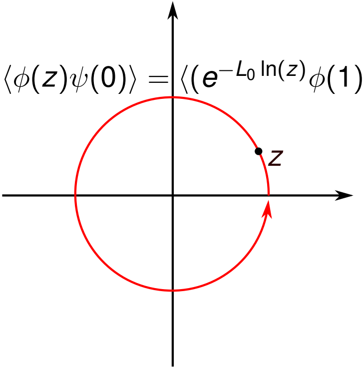
Outline

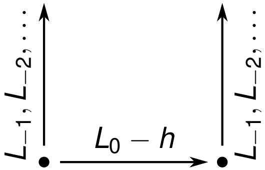
What is logCFT?

Correlators can contain logarithms.

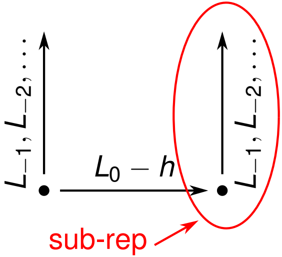
$$\langle \phi(z)\psi(0)\rangle = \langle (e^{-L_0 \ln(z)}\phi(1))\psi(0)\rangle$$



Implication for representations



Implication for representations



Examples

Examples will be constructed similarly to minimal models with central charge

$$c_{p,q} = 1 - 6\frac{(p-q)^2}{pq}, \quad p, q \geq 2,\ (p,q) = 1$$

by free field theories and screening operators.

- The minimal model VOA and representation theory is given by the cohomologies of screening operators [Felder],[Tsuchiya,Kanie]
- Obtain logarithmic VOA by considering only the kernels of screening operators (also allow $p = 1$ or $q = 1$) [Feigin, Gainutdinov, Semikhatov, Tipunin]

This logarithmic VOA is called the $W(p,q)$ triplet.

- generated by T and 3 W-fields at level $(2p-1)(2q-1)$
- $2n-1$ Virarsoro primaries at levels $(np-1)(nq-1)\ \forall n \geq 1$
- reducible for $p \neq 1 \neq q$
- conjectured to be c_2-cofinite (proven for $p = 1, 2$ [Adamovic,Milas], [Nagatomo, Tsuchiya])

So where are the logarithms?

Logarithmic reps appear in fusion of irreds.
How can these fusion products be calculated?

- Verlinde formula leeds to strange results
- From correlators with Ward identities: difficult for algebras larger than just Virasoro
- lattice methods
- NGK algorithm
  1. $W(1,q)$ fusion rules conjectured [Gaberdiel, Kaush],[Gaberdiel, Runkel]
  2. $W(1,q)$ fusion rules proven [Tsuchiya, SW]
  3. $W(p,q)$ fusion rules conjectured [Eberle, Flohr], [Gaberdiel, Runkel, SW], [Pearce, Rasmussen, Ruelle]

Do it yourself fusion

The NGK algorithm [Nahm], [Gaberdiel], [Gaberdiel,Kausch], [Tsuchiya,Hashimoto], [Tsuchiya,SW]:

- Consider fields $\phi_i \in R_i, i = 1, 2, 3$ in a correlator with a holomorphic field (e.g. $T(z)$)

$$\langle \phi_3(\infty) z^{m+1} T(z) \phi_2(1) \phi_1(0) \rangle$$

- Integrate over circle containing 0,1

$$\oint z^{m+1} \langle \phi_3(\infty) T(z) \phi_2(1) \phi_1(0) \rangle dz = \langle \phi_3(\infty) L_m(\phi_2(1) \phi_1(0)) \rangle$$

- Expand at $0, 1, \infty$ to get action at those points.

$$\langle(\Delta_\infty(L_m)\phi_3(\infty))\phi_2(1)\phi_1(0)\rangle = \langle\phi_3(\infty)\Delta_{1,0}(L_m)(\phi_2(1)\phi_1(0))\rangle$$

- If

$$\langle\phi_3(\infty)L_m(\phi_2(1)\phi_1(0))\rangle \neq 0$$

then $\phi_2(1) \otimes \phi_1(0)$ fuses to $\Delta_\infty(L_m)\phi_3(\infty)$.
- This allows us to construct $R_3 = R_2 \otimes_f R_1$.

- Not all states in $R_2 \otimes_{\mathbb{C}} R_1$ lie in $R_2 \otimes_f R_1$.
- Construct order by order through a family of quotients

$$(R_2 \otimes_{\mathbb{C}} R_1)^{(n)} = \Delta_{1,0}(\mathcal{U}[> n])(R_2 \otimes_{\mathbb{C}} R_1)$$

$$\widehat{R_2 \otimes_f R_1} = \varprojlim_n \frac{R_2 \otimes_{\mathbb{C}} R_1}{(R_2 \otimes_{\mathbb{C}} R_1)^{(n)}}$$

$$R_2 \otimes_f R_1 = \bigoplus_{d \in \mathbb{C}} \widehat{R_2 \otimes_f R_1}[d]$$

Pros:

- First principles calculation that derives directly from correlators
- Does not assume semi-simplicity or highest weight reps
- 0th order quotient

$$\frac{R_2 \otimes_{\mathbb{C}} R_1}{(R_2 \otimes_{\mathbb{C}} R_1)^{(0)}}$$

  is easy to compute and contains a lot of information

Cons:

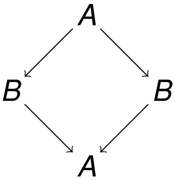
- Higher quotients are tedious to compute
- Inverse limit almost impossible to calculate
- Much harder than Verlinde formula

Theorem (Hashimoto, Tsuchiya)

If the symmetry algebra is c_2 cofinite then the above fusion product defines a braided monoidal structure on the representation category.

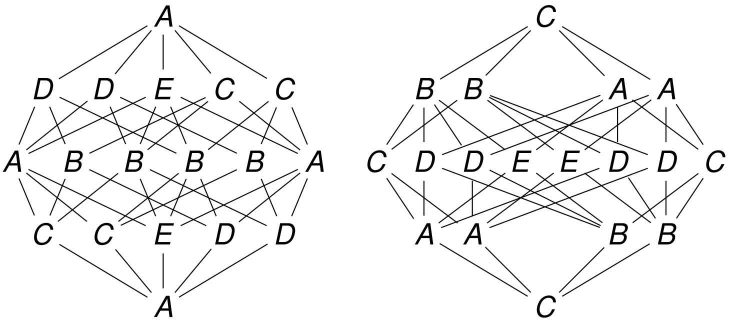
This tensor category structure guarantees associative boundary and bulk algebras when constructing the bulk theory [Fjelstad,Fuchs,Runkel,Schweigert], [Gaberdiel,Runkel,SW].

Simplest logarithmic reps that appear in fusion in terms of *indecomposable combinations* of irreds [Gaberdiel,Kausch]



Verlinde formula leads to problems because it only sees characters! This rep has the same character as $2A \oplus 2B$.

Most complicated logarithmic reps found so far [Gaberdiel, Runkel, SW],[Rasmussen, Pearce],[Adamovic,Milas]



Comparison according to escalating complexity

	minimal	$W(1,q)$	$W(p,q)$
algebra	simple	simple	reducible
rep-thy	semi-simple	not semi-simple	not semi-simple
modular transf	$SL(2,\mathbb{Z})$-rep	τ-factors	τ^2-factors
fusion	exact	exact	not exact
bulk thy	classified	diagonal thys	diagonal thy for (2,3)

Open problems for $W(p, q)$ models

- c_2-cofiniteness
- classification of all irreds
- classification of all indecomposables
- fusion rules
- Verlinde formulae
- Classification of CFTs with $W(p, q)$ symmetry

Long term open problems

- generalise $\kappa = p/q$ to $\kappa \in \mathbb{C}$
- generalise the lattice of the free field construction to higher rank

Conclusion

- Showed some strange properties of logCFT such as indecomposabiltiy, difficulties in computing fusion or problems with modular transformations.
- Claimed that these can sort of be dealt with.
- Thank you for your attention.

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