Superstring Perturbation Theory Revisited

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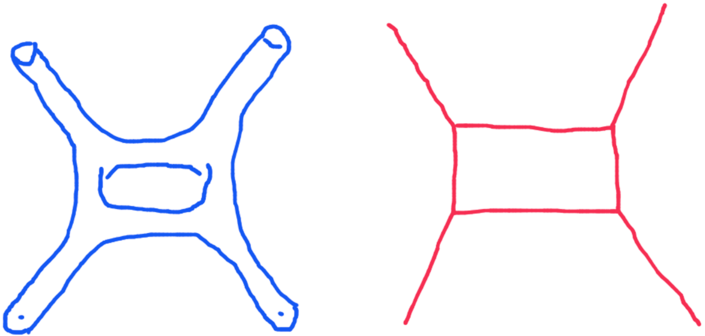

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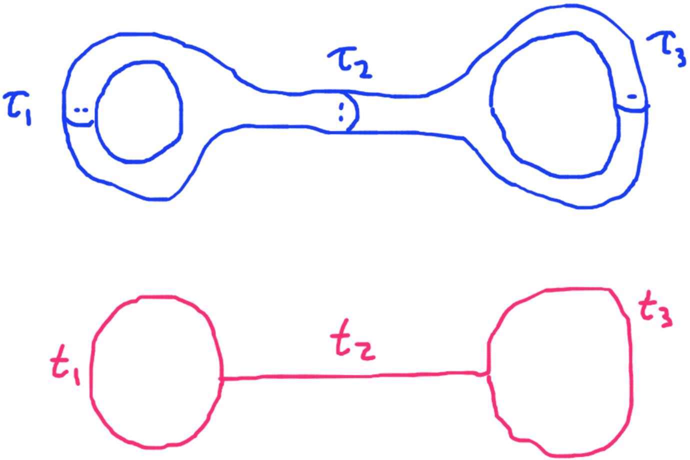

Bosonic string theory – that is string theory with Riemann surfaces – resolves the ultraviolet problems of ordinary quantum field theory, but it has unavoidable infrared problems associated to tachyons and also to "tadpoles" of massless particles.

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Riemann surfaces are certainly very familiar to string theorists, and since complex manifolds of higher dimension are also important in string theory, to some extent string theorists have become algebraic geometers. But super Riemann surfaces have not become so well known, even among string theorists, and the subject has not been so well developed. Partly in consequence, although the key ideas of superstring perturbation theory were well established in the 1980's, some nagging details were never settled.

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The natural way to develop superstring perturbation theory is in terms of super Riemann surface theory. There has been surprisingly little work along these lines, though a celebrated genus 2 calculation by E. D'Hoker and D. H. Phong used this framework, and an approach to a general story was made in a series of remarkable but little-known papers in the 1990's by A. Belopolsky.

A super Riemann surface (with $N = 1$ SUSY) is a supermanifold of dimension $1|1$, but it has much more structure than that (see Rosly, A. Schwarz, and Voronov (1988) for what I will say and much more). There are far too many $1|1$ supermanifolds.

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$$0 \to \mathcal{D} \to T\Sigma \to \mathcal{D}^2 \to 0.$$

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Such coordinates are called superconformal coordinates. The superconformal generators (i.e., the vector fields that preserve $\mathcal{D}$) are then locally of the form $f(z)(\partial_\theta - \theta \partial_z)$ and

$$-(f(z)(\partial_\theta - \theta \partial_z))^2 = g \frac{\partial}{\partial z} + \frac{g'}{2} \theta \frac{\partial}{\partial \theta}, \quad g = f^2,$$

which are familiar formulas.

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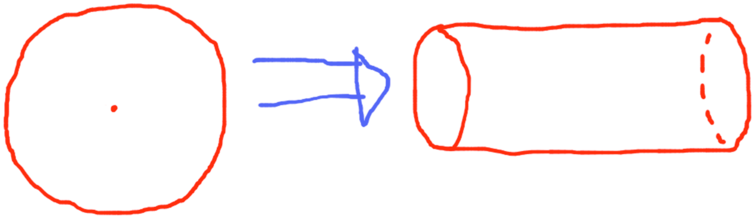

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Accordingly, on a super Riemann surface, a point can sometimes behave like a divisor, just as on an ordinary Riemann surface and unlike the case of a generic $1|1$ supermanifold.

In fact, in superstring theory, there are two kinds of vertex operator. A Neveu-Schwarz vertex operator is a field $\Phi(z, \theta)$ that is inserted at a point $z, \theta \in \Sigma$ and thus what I have said applies directly to such vertex operators.

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$$D = \frac{\partial}{\partial\theta} + z\theta\frac{\partial}{\partial z}$$

so $D^2 = z\partial/\partial z$ and vanishes on the divisor $z = 0$.

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Thus Ramond vertex operators are directly associated to divisors – though this is not usually stated – while NS vertex operators can be associated to divisors via the map from points to divisors on a super Riemann surface.

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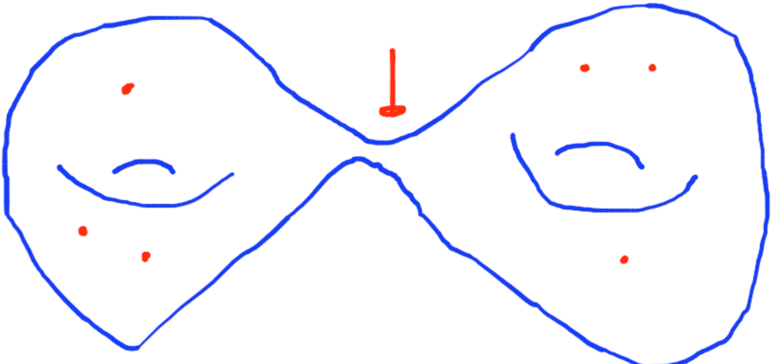
Supermoduli space is not compact – or if we take its Deligne-Mumford compactification, then the measure we want to integrate has singularities – because the infrared singularities that are crucial to the physical interpretation of string theory arise from the behavior of the measure at infinity



Although supermoduli space is very subtle, if one asks precisely the questions whose answers one needs, those particular questions tend to have simple answers.

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For the super case, we have to decide whether the string state propagating through the double point is in the NS or Ramond sector. But either way, there is a formula almost as simple as the bosonic one.

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Importantly, the gluing depends in both cases on only one bosonic parameter ε and no fermionic ones, just as for bosonic Riemann surfaces. The locus $\varepsilon = 0$ in $\widetilde{\mathcal{M}}_{g,n}$ is a product of spaces of the same type $\widetilde{\mathcal{M}}_{g_1,n_1+1} \times \widetilde{\mathcal{M}}_{g_2,n_2+1}$ with $g_1 + g_2 = g$, $n_1 + n_2 = n$, just as for bosonic Riemann surfaces.

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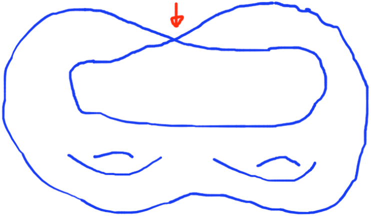
For another example, although a sum over spin structures (independent of the integration over supermoduli) does not make sense in general, a very small piece of it makes sense when a node develops



and this leads to the GSO projection on the physical states that propagate through the node.

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Let me mention at least a few of the important structures in bosonic string theory that one has to generalize to super Riemann surfaces in order to have a good foundation for superstring perturbation theory. If $\mathcal{X}$ is an observable – for example a product $\mathcal{X} = \mathcal{V}_1\mathcal{V}_2 \dots \mathcal{V}_n$ of vertex operators – then there is an associated differential form $F_{\mathcal{X}}$ on $\mathcal{M}_{g,n}$.

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The association $\mathcal{X} \to F_{\mathcal{X}}$ is also compatible with BRST symmetry in the sense that (with Q the BRST operator)

$$F_{Q\mathcal{X}} + \mathrm{d}F_{\mathcal{X}} = 0.$$

This is the basis of the proof of gauge-invariance. Usually one considers a product of physical state vertex operators $\mathcal{V}_1, \ldots, \mathcal{V}_n$, all of them annihilated by Q. One makes a gauge transformation $\mathcal{V}_1 \to \mathcal{V}_1 + \{Q, \mathcal{W}_1\}$, for some $\mathcal{W}_1$. This shifts the scattering amplitude by

$$\int_{\mathcal{M}_{g,n}} F_{\mathcal{V}_1 \mathcal{V}_2 \ldots \mathcal{V}_n} \to \int_{\mathcal{M}_{g,n}} \left(F_{\mathcal{V}_1 \mathcal{V}_2 \ldots \mathcal{V}_n} + F_{Q\mathcal{W}_1 \mathcal{V}_2 \ldots \mathcal{V}_n} \right).$$

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The extra term that should vanish to establish gauge invariance is thus

$$\int_{\mathcal{M}_{g,n}} F_{Q\mathcal{W}_1\,\mathcal{V}_2\ldots\mathcal{V}_n} = -\int_{\mathcal{M}_{g,n}} \mathrm{d}F_{\mathcal{W}_1\mathcal{V}_2\ldots\mathcal{V}_n}$$

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where I used the compatibility of $\mathcal{X} \to F_{\mathcal{X}}$ with BRST symmetry. Finally we have Stokes's theorem

$$\int_{\mathcal{M}_{g,n}} \mathrm{d} F_{\mathcal{W}_1 \mathcal{V}_2 \ldots \mathcal{V}_n} = - \int_{\partial \mathcal{M}_{g,n}} F_{\mathcal{W}_1 \mathcal{V}_2 \ldots \mathcal{V}_n}.$$

Thus gauge-invariance finally depends only on the behavior of the form $F_{\mathcal{W}_1 \mathcal{V}_2 \ldots \mathcal{V}_n}$ in the infrared region, that is at infinity in $\mathcal{M}_{g,n}$.

So this is the package that one needs to carry over to super Riemann surfaces in order to have a proper foundation for superstring perturbation theory. One needs an association $\mathcal{X} \to F_{\mathcal{X}}$ of observables to forms on moduli space, which maps ghost number to degree and maps the BRST operator Q to the exterior derivative d. And one needs Stokes's theorem so that one can integrate by parts.

So this is the package that one needs to carry over to super Riemann surfaces in order to have a proper foundation for superstring perturbation theory. One needs an association $\mathcal{X} \to F_{\mathcal{X}}$ of observables to forms on moduli space, which maps ghost number to degree and maps the BRST operator Q to the exterior derivative d. And one needs Stokes's theorem so that one can integrate by parts. It turns out that when one works out what these things mean in supergeometry, one meets another structure, picture number, which was part of the framework of Friedan, Martinec, and Shenker (and then was interpreted more geometrically by E. and H. Verlinde and then by Belopolsky).

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It is not really possible to explain everything in one lecture, so perhaps I will focus on explaining the notion of forms on a supermanifold, picture number, and Stokes's theorem. Suppose that M is a bosonic manifold with local coordinates $x^1, \dots, x^n$. We let ΠTM be the cotangent bundle with "parity" (or statistics) reversed on the fibers, so local coordinates on ΠTM are $x^1, \dots, x^n$ and corresponding fermionic variables that we will call $\mathrm{d}x^1, \dots, \mathrm{d}x^n$.

A function on ΠTM can be expanded in powers of the $\mathrm{d}x$'s

$$f(x^1,\ldots,x^n|\mathrm{d}x^1\ldots\mathrm{d}x^n) = f_0(x^1\ldots x^n) + \sum_i \mathrm{d}x^i f_{1,i}(x^1,\ldots,x^n)$$

$$+\sum_{i<j}\mathrm{d}x^i\mathrm{d}x^j f_{2,ij}(x^1,\ldots,x^n)+\ldots.$$

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If $f(x|\mathrm{d}x)$ is homogeneous of degree k in the $\mathrm{d}x$'s, it is usually called a k-form.

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So for example if

$$f(x, \mathrm{d}x) = \cdots + \mathrm{d}x^1 \mathrm{d}x^2 \ldots \mathrm{d}x^n \, f_{(n)}(x^1, \ldots, x^n),$$

where I have only written the n-form part of f, then the Berezin integral over the $\mathrm{d}x^i$ picks out the n-form part of f

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$$\int_M f(x, \mathrm{d}x) = \int_{\Pi TM} \mathcal{D}(x, \mathrm{d}x) \, f(x, \mathrm{d}x).$$

On ΠTM, there is a vector field of degree 1

$$\mathrm{d} = \sum_{i=1}^{n} \mathrm{d}x^i \frac{\partial}{\partial x^i}$$

and Stokes's theorem says that

$$\int_M \mathrm{d}g = \int_{\partial M} g.$$

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However, there is a key difference from the bosonic case: there are different classes of functions on ΠTM. For an ordinary manifold M, the fiber coordinates $\mathrm{d}x^i$ were fermionic variables, so any function on ΠTM was a polynomial along the fibers. We did not have to choose a class of functions.

However, there is a key difference from the bosonic case: there are different classes of functions on ΠTM. For an ordinary manifold M, the fiber coordinates $\mathrm{d}x^i$ were fermionic variables, so any function on ΠTM was a polynomial along the fibers. We did not have to choose a class of functions. For $M = \mathbb{R}^{0,n}$, the fiber coordinates are even and there definitely are different classes of functions on ΠTM.

For example, we can consider functions on ΠTM that are polynomials in the $\mathrm{d}\theta$'s. These functions are called differential forms.

For example, we can consider functions on ΠTM that are polynomials in the $\mathrm{d}\theta$'s. These functions are called differential forms. They are closed under many natural operations, but they cannot be integrated, because obviously, with $\mathrm{d}\theta$ being an ordinary even variable, an integral

$$\int \mathcal{D}(\theta, \mathrm{d}\theta)\, f(\theta, \mathrm{d}\theta)$$

diverges if $f(\theta, \mathrm{d}\theta)$ is a polynomial in $\mathrm{d}\theta$.

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$$g(\theta, \mathrm{d}\theta) = g_{-1}(\theta)\delta(\mathrm{d}\theta) + g_{-2}(\theta)\delta'(\mathrm{d}\theta) + g_{-3}(\theta)\delta^{(2)}(\mathrm{d}\theta) + \dots.$$

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Concretely the integral over $\mathrm{d}\theta$ picks out the top form g_{-1} and so

$$\int_M g(\theta, \mathrm{d}\theta) = \int \mathcal{D}(\theta)\, g_{-1}(\theta)$$

where the last integral is a Berezin integral.

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For the case of a purely fermionic supermanifold $\mathbb{R}^{0|n}$, Stokes's theorem says that for any integral form g,

$$\int_M \mathrm{d}g = 0.$$

For instance, if

$$g(\theta, \mathrm{d}\theta) = g_{-1}(\theta)\delta(\mathrm{d}\theta) + g_{-2}(\theta)\delta'(\mathrm{d}\theta) + g_{-3}(\theta)\delta^{(2)}(\mathrm{d}\theta) + \dots,$$

then

$$\mathrm{d}g = -\frac{\partial}{\partial\theta} g_{-2}(\theta)\delta(\mathrm{d}\theta) + \dots$$

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The general supermanifold version of Stokes's theorem is more or less a combination of this with the ordinary bosonic Stokes's theorem.

I have described differential forms (polynomial dependence on all $\mathrm{d}\theta$'s) and integral forms (delta function dependence on all $\mathrm{d}\theta$'s. More generally, one considers other classes of forms that are closed under various natural operations (such as scaling of $\mathrm{d}\theta$, multiplication by $\mathrm{d}\theta$, and differentiation $\partial/\partial(\mathrm{d}\theta)$).

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Obviously we can't go into a full explanation today, but it turns out that superstring perturbation theory is nicely compatible with this formalism and this gives a natural framework to understand gauge-invariance, spacetime supersymmetry, and tadpole cancellation.

The traditional alternative is to look for a map from $\widetilde{\mathcal{M}}_{g,n}$ to $\mathcal{M}_{g,n}$ (a map that is the identity when restricted to $\mathcal{M}_{g,n} \subset \widetilde{\mathcal{M}}_{g,n}$) and try to formulate everything in terms of measures on $\mathcal{M}_{g,n}$.

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