Workshop

Random Matrices

Hausdorff Center for Mathematics
University of Bonn

May 29 – June 1, 2012

Organizers
Patrik Ferrari (University of Bonn)
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Program Overview

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Random Matrices

Random Matrices and their analysis play an important role in various areas, such as mathematical physics, statistics, Banach space geometry, signal processing (compressive sensing), analysis of optimization algorithms, growth models and more. The interaction with application fields, in particular, has triggered high research activity in random matrix theory recently. The proposed workshop aims at bringing together experts and junior researchers working on various aspects of random matrices, and to report on recent advances. In particular, we aim at identifying possible new directions and methods that may arise from the combination of different expertises.

Some Infos

Location

All lectures of the workshop will take place at the Mathematics Center, Endenicher Allee 60, in the Lipschitz Lecture Room, first floor (European counting).

Welcome Reception on Wednesday

On Wednesday evening there will be a welcome reception starting at 18:00 in the Plücker room (next to the Lipschitz Lecture Room) at the Mathematics Center, Endenicher Allee 60, Bonn.

Excursion on Thursday

On Thursday afternoon we plan an excursion starting at 15:00 right after the last talk. We will walk to the Rhine river and take a boat to the city of Königswinter. From there we will hike up to the famous castle Drachenfels (about 1 hour hike), where you will have a beautiful view over the Rhine and the surrounding area. On our way back we can either take the cable car down the hill and have dinner near the Rhine river in Königswinter, or walk down and have dinner half-way down in some nice beer garden with a beautiful view. We will return to Bonn by tram.

The transportation cost for the excursion will be about 10 Euro. Please let us know whether you would like to join the excursion. There will be a sign-up list at the reception on Tuesday.

WiFi

All participants should have received information (including some necessary files) via e-mail on how to log on to the WiFi in the Mathematics Center. Please check your old mails (around May 20).

Alternatively, one can log on to the network *MIgast* with a pair of user name and password available at the registration. This second option is simpler to use, but the connection is not encrypted and therefore less secure.

Library

Please feel free to use the mathematics library in Endenicher Allee 60. It contains a large collection of books and journals. The entrance to the library is on the ground floor.

Discussion Rooms

We have a number of rooms available for individual discussions. Please ask one of the organizers. In addition, there are "discussion tables" distributed in the Mathematics Center, near the stairways in both sidewings of the building.

Acknowledgements

The organizers would like to thank the Hausdorff Center for Mathematics for the generous financial support, which made this workshop possible. In addition, we acknowledge funding from the European Research Council.

Special thanks go to Laura Siklossy for the great support in all administrative issues, and to Monika Brüggemann and her team for organizing the coffee breaks and the reception. We further acknowledge the efforts by Ulaş Ayaz, Nils Benedikter, Sjoerd Dirksen, René Frings, Max Hügel, Maryia Kabanava, Peter Nejjar, Gunder Sievert, and Zeljka Stojanac for helping with practical issues during the workshop and for producing this booklet.

Program

Tuesday, May 29

8:30 - 9:00 Registration

9.00 - 9.10 Welcome

9:10 - 9:55 Kurt Johansson
The tacnode process

9:55 - 10:40 Eric Nordenstam
Tilings of half a hexagon

10:40 - 11:15 *Coffee Break*

11:15 - 12:00 Gernot Akemann
Wilson random matrix theory and the chGUE - GUE transition

12:00 - 14:00 *Lunch Break*

14:00 - 14:45 Horng-Tzer Yau
Local circular law for random matrices

14:55 - 15:40 Laszlo Erdős
Diffusion profile and delocalization for random band matrices

15:40 - 16:15 *Coffee Break*

16:15 - 17:00 Antti Knowles
Spectral statistics of deformed Wigner matrices

Wednesday, May 30

9:00 - 9:45 Alain Pajor
Weak and strong moment estimates for convex measures

9:55 - 10:40 Roman Vershynin
Invertibility of symmetric random matrices

10:40 - 11:15 *Coffee break and Poster Session*

11:15 - 12:30 *Poster Session*

12:30 - 14:00 *Lunch Break*

14:00 - 14:45 Alexander Soshnikov
Outliers in large random matrices

14:55 - 15:40 Sandrine Péché
Heavy tailed random matrices

15:40 - 16:15 *Coffee Break*

16:15 - 17:00 Eugene Kanzieper
RMT and quantum transport through chaotic cavities: New results to the old problem

17:10 - 17:55 Jon Warren
A multilayer extension of the KPZ equation

18:00 *Reception*

Thursday, June 1

9:00 - 9:45 Alice Guionnet
Localization and delocalization of eigenvectors for heavy-tailed random matrices

9:55 - 10:40 Felix Krahmer
Suprema of chaos processes and the restricted isometry property

10:40 - 11:15 *Coffee break*

11:15 - 12:00 Nicole Tomczak-Jaegermann
Recent results on log-concave ensemble

12:00 - 14:00 *Lunch Break*

14:00 - 14:45 Mark Rudelson
The smallest singular value of a unitary perturbed matrix

15.00 *Excursion*

Friday, June 1

9:00 - 9:45 Folkmar Bornemann
Accurate numerical computations for classical matrix ensembles

9:55 - 10:40 Sasha Sodin
A 2D gradient model with non-convex interaction

10:40 - 11:15 *Coffee break*

11:15 - 12:00 Holger Kösters
Quadratic forms of inverse covariance matrices

12:00 - 14:00 *Lunch Break*

14:00 - 14:45 Stanislaw Szarek
Phase transitions for high-dimensional random quantum states

14:55 - 15:40 Herbert Spohn
Random matrices and the KPZ equation

15:40 - 16:15 *Coffee*

Abstracts

Tuesday, 9.10-9.55
The tacnode process

Kurt Johansson
University of Stockholm

The tacnode process is a a determinantal point process that occurs as the scaling limit in several models where two Airy processes "collide". It has been studied recently by Adler, Ferrari and Van Moerbeke, Delvaux, by Kuijlaars and Zhang and by myself. I will discuss the occurrence of this limiting process both in a Brownian motion model and in a random tiling model called the Double Aztec diamond.

Tuesday, 9.55-10.40
Tilings of half a hexagon

Eric Nordenstam
University of Vienna

We study tilings of a certain region which we call the Novak half hexagon. It turns out that the number of tilings with rhombuses of a size n Novak half hexagon is exactly the number of tilings with dominoes of a size n Aztec diamond, which is a tiling model that is extremely well studied in the random matrix community. In our failed attempt to find a combinatorial bijection between these objects, we nevertheless found many interesting similarities between these two models. There is an algorithm to sample tilings of the Novak half hexagon which is very reminiscent to the shuffling algorithm for the Aztec diamond. The arctic circle theorem for the Aztec diamond can be used to prove an arctic parabola theorem for the Novak half hexagon. There is also a determinantal point process associated to this model for which we derive the kernel.

Tuesday, 11.15-12.00
Wilson random matrix theory and the chGUE-GUE transition

Gernot Akemann
University of Bielefeld

In this talk I will present the solution of a two-matrix model that describes spectral properties of the Wilson Dirac operator in particle physics. The simplest version of this matrix model corresponds to the transition between the symmetry class of the chiral Gaussian Unitary Ensembles (chGUE) and the GUE. All eigenvalue density correlation functions of the Hermitian Wilson Dirac operator can be expressed in terms of a matrix kernel of skew orthogonal polynomials, given by an integral transform of the classical Laguerre polynomials. In the microscopic large-N limit close to the chGUE symmetry class interesting properties are found including factorisation. We also find a Fredholm like expansion for the gap probabilities even when the matrix model is not a determinantal or Pfaffian point process.

Tuesday, 14.00-14.45
Local circular law for random matrices

Horng-Tzer Yau
Harvard University

Tuesday, 14.55-15.40

Diffusion profile and delocalization for random band matrices

Laszlo Erdös
TU Munich

We consider Hermitian random band matrices H in $d \geq 1$ dimensions. The matrix elements h_{xy}, indexed by a box of linear size L in the d-dimensional lattice, are independent, centred random variables with variances $s_{xy} = \mathbb{E}|h_{xy}|^2$. We assume that s_{xy} becomes negligible as $|x-y|$ exceeds a given parameter W, the band width. We derive a new self-consistent equation for the averaged matrix elements $\mathbb{E}|G_{xy}|^2$ of the resolvent $G = G(z) = (H-z)^{-1}$ and we prove that they are given by an effective diffusion operator in a certain regime of the parameters. As a corollary we show that in $d = 1$ dimensions most eigenfunctions are delocalized if $W \geq L^{4/5}$. Similar results are obtained in higher dimensions, $d > 1$. This is a joint work with A. Knowles, H.-T. Yau and J. Yin.

Tuesday, 16.15-17.00

Spectral statistics of deformed Wigner matrices

Antti Knowles
Harvard University

I report on recent joint work with Jun Yin, in which we consider Wigner matrices that have been deformed by the addition of a finite-rank matrix. By Weyl's interlacing inequalities, this deformation does not affect the large-scale statistics of the spectrum. However, it may affect eigenvalues near the spectral edge, causing them to break free from the bulk spectrum, hence becoming "outliers". We prove that the non-outliers stick to eigenvalues of the original Wigner matrix, and identify the asymptotic distribution of the outliers. In particular, we obtain a dynamical picture of repulsion among the outliers, as well as a complete description of the failure of universality.

Wednesday, 9.00-9.45

Weak and strong moment estimates for convex measures

Alain Pajor
University of Paris Est Marne-la-Valle

We discuss the comparison of weak and strong moments of random vectors in an Euclidean space, whose distribution is a convex measure (typically with a density of the form $1/f^\beta$ with f convex positive). We will survey recents results in collaboration with Radoslaw Adamczak, Olivier Guedon, Rafal Latala, Alexander Litvak, Krzysztof Oleszkiewicz and Nicole Tomczak-Jaegermann.

Wednesday, 9.55-10.40

Invertibility of symmetric random matrices

Roman Vershynin
University of Michigan

We will discuss invertibility of general $N \times N$ Wigner and Wishart random matrices, possibly discrete. We address two questions: (a) What is the probability that the random matrix is singular? (b) What is the typical norm of the inverse? We discuss several recent results in this area. A general Wigner matrix W is (a) singular with exponentially small probability $\exp(-N^{const})$, and (b) the norm of the inverse is $\mathcal{O}(\sqrt{N})$. The eigenvalues and eigenvectors are well delocalized. Regarding Wishart matrices with p degrees of freedom, they are well invertible for $n = \mathcal{O}(p \log p)$ (this is a 1999 result of Rudelson), and the logarithmic factor can be removed for "most" distributions – those with $2 + o(1)$ bounded moments. The latter is a joint work joint with Srivastava based on randomizing the spectral sparsification method of Batson, Spielman and Srivastava.

Wednesday, 14.00-14.45
Outliers in large random matrices

Alexander Soshnikov
UC Davis

We discuss the fluctuation of matrix entries of regular functions of Wigner random matrices and the related results about the fluctuation of the outliers of finite rank deformations of Wigner matrices.

The talk is based on the results of four papers arXiv:1103.3731 math.PR (to appear in Annales I.H.P. (B) Probab. et Stat), arXiv:1103.1170 math.PR (J. Stat. Phys. v. 146, 2012), arXiv:1104.1663 math.PR (to appear in J. Theor. Probab.), and arXiv:1203.5130 math.PR, written in collaboration with Sean O'Rourke, Alessandro Pizzo, and David Renfrew.

Wednesday, 14.55-15.40
Heavy-tailed random matrices

Sandrine Péché
University of Grenoble

We will review the known/unknown results about "heavy tailed" random matrices in the large N limit. We will focus on the spectrum of the usual random matrix ensembles under the assumption that the entries have few finite moments.

Wednesday, 16.15-17.00

RMT and quantum transport through chaotic cavities: New results to the old problem

Eugene Kanzieper
Holon Institute of Technology

In this talk I review recent random-matrix-theory results for the paradigmatic problem of universal quantum transport through chaotic cavities coupled to the outside world through both ballistic and tunnel point contacts. (a) In case of ballistic point contacts (ideal leads), the ideas of integrability will be utilised to demonstrate that fluctuations of the Landauer conductance are governed by the fifth Painlevé transcendent and a one-dimensional Toda lattice equation. (b) Further, inclusion of tunnel effects (switching from ideal to non-ideal leads) will be shown to modify a joint probability density function of reflection eigenvalues whose derivation will briefly be outlined. I shall use the latter to argue that in case of non-ideal leads the fluctuations properties of Landauer conductance are governed by a two-dimensional Toda lattice equation. The talk is based on a joint work with Vladimir Al. Osipov and Pedro Vidal.

Wednesday, 17.10-17.55

A multilayer extension of the KPZ equation

Jon Warren
University of Warwick

The Airy process describes the evolution of the largest eigenvalue of Hermitian Brownian motion, and extends to the so called multiline Airy process that describes the k-th largest eigenvalue for $k = 1, 2, 3 \ldots$ We know that the solution to the KPZ equation, starting from a "wedge" initial condition converges, as time goes to infinity, to the Airy process, and it is natural to try to construct a multiline extension of the KPZ. This will be described in the talk, along with various properties of the resulting object.

Thursday, 9.00-9.45

Localization and delocalization of eigenvectors for heavy-tailed random matrices

Alice Guionnet
University of Lyon

Consider an $n \times n$ Hermitian random matrix with, above the diagonal, independent entries with α-stable symmetric distribution and $0 < \alpha < 2$. We establish new bounds on the rate of convergence of the empirical spectral distribution of this random matrix as n goes to infinity. When $1 < \alpha < 2$ and $p > 2$, we give vanishing bounds on the L^p-norm of the eigenvectors normalized to have unit L^2-norm. On the contrary, when $0 < \alpha < 2/3$, we prove that these eigenvectors are localized.

Joint work with Charles Bordenave.

Thursday, 9.55-10.40

Suprema of chaos processes and the restricted isometry property

Felix Krahmer
University of Göttingen

The theory of compressed sensing considers the following problem: Let $A \in \mathbb{C}^{m \times n}$ and let $x \in \mathbb{C}^n$ be s-sparse, i.e., $x_i = 0$ for all but s indices i. One seeks to recover x uniquely and efficiently from linear measurements $y = Ax$, although $m \ll n$. A sufficient condition to ensure that this is possible is the Restricted Isometry Property (RIP). A is said to have the RIP, if its restriction to any small subset of the columns acts almost like an isometry.

In this talk, we study two classes of matrices with respect to the RIP: First, we consider matrices A which represent the convolution with a random vector followed by a restriction to an arbitrary fixed set of entries. We focus on the scenario that ϵ is a Rademacher vector, i.e., a vector whose entries are independent random signs. Second, we study Gabor synthesis matrices, that is, matrices consisting of time-frequency shifts of a Rademacher vector.

In both cases, this question can be reduced to estimating random variables of the form

$$D_{\mathcal{A}} := \sup_{A \in \mathcal{A}} \left| \|A\epsilon\|^2 - \mathbb{E}\|A\epsilon\|^2 \right|,$$

where $\mathcal{A}$ is a set of matrices. Random variables of this type are closely related to suprema of chaos processes. Using generic chaining techniques, we derive

a bound for $\mathbb{E}D_{\mathcal{A}}$ in terms of the Talagrand γ_2-functional. As a consequence, we obtain that matrices from both classes under consideration have the RIP with high probability if the embedding dimension satisfies $m \geq Cs\log(n)^4$. This bound exhibits optimal dependence on s, while previous works had only obtained a suboptimal scaling of $s^{3/2}$.

This is joint work with Shahar Mendelson and Holger Rauhut.

Thursday, 11.15-12.00

Recent results on log-concave ensemble

Nicole Tomczak-Jaegermann
University of Alberta

We describe new tail estimates for norms of projections of sums of independent log-concave random vectors in $\mathbb{R}^n$; and their uniform versions in forms of operator norms of matrices and submatrices of independent log-concave rows or columns.

The talk is based on a number of joint results by (subsets of) the following authors: Radoslaw Adamczak, Olivier Guedon, Rafał Latała, Alexander Litvak, Krzysztof Oleszkiewicz, Alain Pajor, and the speaker.

Thursday, 14.00-14.45

The smallest singular value of a unitary perturbed matrix

Mark Rudelson
University of Michigan

We study the distribution of the smallest singular value of the matrix $M_n = D_n + U_n$, where D_n is an $n \times n$ deterministic matrix, and U_n is a random unitary matrix, uniformly distributed with respect to the Haar measure. A bound for this singular value arises as a condition in the Single Ring Theorem of Guionnet, Krishnapour, and Zeitouni. Consider a family of random matrices with given distributions of singular values. The Single Ring Theorem asserts that under some conditions the empirical distributions of eigenvalues converge to a limit density, supported in a single ring. The conditions are of two types: "scalar", which pertain to the original distribution of singular values, and "matrix", which is significantly harder to check. Our result shows that the condition of the second type is redundant.

Joint work with Roman Vershynin.

Friday, 9.00-9.45

Accurate numerical computations for classical matrix ensembles

Folkmar Bornemann
TU Munich

The highly accurate numerical evaluation of higher order gap probabilities and k-th largest eigenvalue distributions can be based, for the unitary ensembles (UE), on representations in terms of derivatives of Fredholm determinants (which are numerically far more convenient than Painlevé representations). Though formulae in terms of matrix-operator determinants are available for the orthogonal (OE) and symplectic (SE) ensembles, they suffer from regularity and complexity issues. We show how the Forrester-Rains interrelations of OE, SE, and UE can be used to circumvent these problems.

Friday, 9.55-10.40

A 2D gradient model with non-convex interaction

Sasha Sodin
Tel Aviv University

The talk will be about a statistical mechanics model on the 2D lattice. We shall define it, and discuss the connection to height models, the six-vertex model, and dipole gas (all of these will be defined as well). Also, we shall discuss the conjectured phase diagram, and explain which parts are rigorously justified.

Based on discussions with David Brydges and Tom Spencer.

Friday, 11.15-12.00

Quadratic forms of inverse covariance matrices

Holger Kösters
University of Bielefeld

Several classical procedures in multivariate statistics are based on quadratic forms of (regularized) inverses of sample covariance matrices. It seems natural to ask how these procedures perform in situations (frequently encountered in modern statistics) where both the number and the dimension of the observations are large.

In this talk I will present some related results from the perspective of random matrix theory. Particular emphasis will be placed on the effects of the underlying distributional assumptions.

This is based on joint work with Noureddine El Karoui.

Friday, 14.00-14.45

Phase transitions for high-dimensional random quantum states

Stanislaw Szarek
Case Western Reserve University

Quantum systems exhibit features that can not be explained via classical probability. Arguably the most notorious of such features is entanglement, which brings about experimentally verifiable correlations between quantities that are inconsistent with the classical theory. In this talk we present results on emergence of entanglement in random quantum systems. A sample consequence of our findings is as follows. For a system of N identical particles in a random pure state, there is a threshold $K = K(N) \sim N/5$ such that two subsystems of k particles each typically share entanglement if $k > K$, and typically do not share entanglement if $k < K$. By "random" we mean here "uniformly distributed on the sphere of the corresponding Hilbert space." Our approach allows to similarly analyze other properties such as for example positive partial transpose (PPT). Mathematically, each problem reduces to studying properties (albeit somewhat exotic) of high-dimensional complex Wishart matrices. The arguments rely on high-dimensional probability, classical convexity, random matrices, and geometry of Banach spaces. Joint work with G. Aubrun and D. Ye. An overview of the results can be found in arxiv:1112.4582

Friday, 14.55-15.40

Random matrices and the KPZ equation

Herbert Spohn
TU Munich

More than 10 years ago, Baik, Deift, and Johansson noted that, in certain combinatorial problems and in growth models, edge distributions from random matrix theory appear in limit theorems. We have now a much deepened understanding why this is the case. In particular, there are other models, like the one-dimensional KPZ equation, were a similar structure has been unravelled.