Sparse Fourier Approximation in High Dimensions

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The Goal: Sparse Signal Recovery

Frequency Sparse Signal

$5 \sin(x) + \sin(100^\circ x)$

$x$
Why Fourier Sparse?

Motivated by
Applications and problems in function learning, interpolation, and numerical methods...

- Medical Imaging [Lustig et al., 2007] in 3D.
- Function Learning: Boolean circuits, Complexity, Property Testing, and more... [Mansour, Matulef, Akavia, ...]
- Streaming algorithms, Massive Datasets [Gilbert, Indyk, Muthukrishnan, Strauss, ...]
- Faster approximate FFTs: Numerical methods for multiscale problems [Daubechies et al., 2007]
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Problem Setup

Recover $f : [0, 2\pi]^D \rightarrow \mathbb{C}$ well approximated by $k$ terms in the $D$-dimensional tensor product basis of Trigonometric polynomials

$$f(\vec{x}) \approx \sum_{j=1}^{k} \hat{f}(\vec{\omega}_j) \cdot e^{i \vec{x} \cdot \vec{\omega}_j}, \quad \Omega = \{\vec{\omega}_1, \ldots, \vec{\omega}_k\} \subset \left(\frac{N}{2}, \frac{N}{2}\right)^D \cap \mathbb{Z}^D$$

- Use as few samples from $f$ as possible.
- Obtain strong approximation error guarantees.
- Runtime polynomial in $k$, $D$, and $\log N$. 
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We will consider $\hat{f}$ to be in $l^1$
- $\tilde{\hat{f}}$, a vector of the lowest modes in $\mathbb{C}^{N^D}$
- $\tilde{\hat{f}}$, a sequence consisting of $\hat{f}$ followed by zeros
- $\hat{f}^{\text{opt}_k}$, an optimal $k$-term approximation to $\hat{f}$

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Notation

Recover

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Deterministic Result

Suppose $f : [0, 2\pi]^D \mapsto \mathbb{C}$ has $\hat{f} \in l^1$ for $N, D, k, \epsilon^{-1} \in \mathbb{N}$. Then, a simple algorithm can output an $\tilde{x}_S \in \mathbb{C}^{N^D}$ satisfying

$$\| \hat{f} - \tilde{x}_S \|_2 \leq \| \hat{f} - \hat{f}_{k}^\text{opt} \|_2 + \frac{\epsilon \cdot \| \hat{f} - \hat{f}_{(k/\epsilon)}^\text{opt} \|_1}{\sqrt{k}} + 22\sqrt{k} \cdot \| \hat{f} - \tilde{f} \|_1.$$

The runtime as well as the number of function evaluations of $f$ are both

$$O \left( \frac{k^2 \cdot D^4 \cdot \log^4 N}{\epsilon^2 \cdot \log D} \right).$$

- The recovery method requires only sorting, FFTs, and the Euclidean algorithm as subcomponents.
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Random Nonuniform Result

Suppose $f : [0, 2\pi]^D \mapsto \mathbb{C}$ has $\hat{f} \in l^1$ for $N, D, k, \epsilon^{-1} \in \mathbb{N}$. Then, a simple algorithm can output an $\tilde{x}_S \in \mathbb{C}^{ND}$ satisfying

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\|\tilde{f} - \tilde{x}_S\|_2 \leq \|\tilde{f} - \tilde{f}_k^{\text{opt}}\|_2 + \frac{\epsilon \cdot \|\tilde{f} - \tilde{f}_k^{(k/\epsilon)}\|_1}{\sqrt{k}} + 22 \sqrt{k} \cdot \|\tilde{f} - \tilde{f}\|_1
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with nonuniform probability greater than $1 - \frac{1}{N^c}$. The runtime as well as the number of function evaluations of $f$ are both

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O \left( \frac{k \cdot D^4 \cdot \log^4 N \cdot \log \left( \frac{k \log N}{\epsilon} \right)}{\epsilon \cdot \log \log N} \right).
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\left\| \hat{f} - \tilde{x}_S \right\|_2 \leq \left\| \hat{f} - \hat{f}^{\text{opt}}_k \right\|_2 + \frac{\epsilon \cdot \left\| \hat{f} - \hat{f}^{\text{opt}}_{(k/\epsilon)} \right\|_1}{\sqrt{k}} + 22 \sqrt{k} \cdot \left\| \hat{f} - \bar{\hat{f}} \right\|_1
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Proof Overview

1. Develop good binary measurement matrices, $\mathcal{M} \in \{0, 1\}^{m \times N}$, with both analytic and combinatorial structure.

2. Develop fast recovery methods for $f : [0, 2\pi] \mapsto \mathbb{C}$ by utilizing the combinatorial structure of $\mathcal{M}$.

3. Map $f : [0, 2\pi]^D \mapsto \mathbb{C}$ to a one-dimensional function.
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Extending to Many Dimensions

Sample $f^{\text{new}}(x) = f\left(x \cdot \frac{\tilde{N}}{P_1}, \ldots, x \cdot \frac{\tilde{N}}{P_D}\right)$, with $\tilde{N} = \prod_{d=1}^{D} P_d > N^D$

- Works because $\mathbb{Z}_{\tilde{N}}$ is homomorphic to $\mathbb{Z}_{P_1} \times \cdots \times \mathbb{Z}_{P_D}$. 
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Incoherent Discrete Matrices

Discrete Incoherence

Let $K, \alpha \in [1, N] \cap \mathbb{N}$. Call an $m \times N$ boolean matrix, $\mathcal{M} \in \{0, 1\}^{m \times N}$, $(K, \alpha)$-coherent if both of the following hold:

1. Every column of $\mathcal{M}$ contains at least $K$ nonzero entries.
2. For all $j, l \in [0, N)$ with $j \neq l$, the associated columns, $\mathcal{M}_{., j}$ and $\mathcal{M}_{., l} \in \{0, 1\}^m$, have $\langle \mathcal{M}_{., j}, \mathcal{M}_{., l} \rangle \leq \alpha$.

A $(K, \alpha)$-coherent matrix also is...

- RIP (i.e., preserves $k$-sparse $\vec{x} \in \mathbb{C}^N$ with $\delta_k = (k - 1)\alpha/K$)
- The adjacency matrix of a $(k, K, (k - 1)\alpha/2K)$-unbalanced expander graph $\iff l^1$-RIP
- $k$-strongly selective $\iff$ $k$-disjunct, and $k$-majority selective
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A \((K, \alpha)\)-coherent matrix also is...

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- The adjacency matrix of a \((k, K, (k - 1)\alpha/2K)\)-unbalanced expander graph \( \iff l^1\)-RIP
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Theorem

Let \( n, k \in [0, N) \cap \mathbb{N}, \epsilon^{-1} \in \mathbb{N}^+, \) and \( \tilde{x} \in \mathbb{C}^N. \) Suppose \( M \in \{0, 1\}^{m \times N} \) is a \((K, \alpha)\)-coherent matrix. Then, more than half of the at least \( K \) rows of \( M \) with nonzero entries in \( M \cdot n \) will produce an entry in \( M \tilde{x} \in \mathbb{C}^m \) that estimates \( \tilde{x}_n \) to within \( \frac{\epsilon \cdot \| \tilde{x} - \tilde{x}_{\text{opt}}(k/\epsilon) \|_1}{k} \) precision.

- The median of the entries of \( M \tilde{x} \in \mathbb{C}^m \) produced by rows with 1 in \( M \cdot n \) will have stated the accuracy.
- The structure of the measurement matrix allows standard fast recovery techniques to be employed.
Theorem

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Examples of Incoherent Discrete Matrices

Incoherent Discrete Matrices Include...

- Random Bernoulli Matrices
- Algebraic Constructions (e.g., DeVore’s deterministic RIP matrix)
- Number Theoretic
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Number Theoretic Constructions

\[ \begin{array}{cccccc}
\mathbf{n} & \in & [0, N) & 0 & 1 & 2 \\
\mathbf{n} \equiv 0 \mod 2 & & (1 & 0 & 1 & 0 & 1 & \ldots) \\
\mathbf{n} \equiv 1 \mod 2 & & 0 & 1 & 0 & 1 & 0 & \ldots \\
\mathbf{n} \equiv 0 \mod 3 & & 1 & 0 & 0 & 1 & 0 & \ldots \\
\mathbf{n} \equiv 1 \mod 3 & & 0 & 1 & 0 & 0 & 1 & \ldots \\
\mathbf{n} \equiv 2 \mod 3 & & 0 & 0 & 1 & 0 & 0 & \ldots \\
\vdots & & \vdots & & \vdots & & \vdots & & \vdots \\
\mathbf{n} \equiv 1 \mod 5 & & 0 & 1 & 0 & 0 & 0 & 1 & \ldots \\
\vdots & & \vdots & & \vdots & & \vdots & & \vdots \\
\end{array} \]

- Let \( p_l \) by the \( l \)th prime number, and \( p_{q-1} \leq k \leq p_q \)
- \( K \) primes larger than \( q \) produce \((K, \lfloor \log_{p_q} N \rfloor)\)-coherent matrix
- Product with Discrete Fourier Transform Matrix is very sparse
- Number of rows of \( \mathcal{M} = \text{Rows of } \mathcal{M} \mathcal{F} \) with nonzero entries =

\[
\sum_{j=0}^{K-1} p_{q+j} \leq \frac{7k^2 \cdot \lfloor \log_k N \rfloor^2}{\epsilon^2} \ln \left( \frac{3.05 \cdot k \cdot \lfloor \log_k N \rfloor}{\epsilon} \right)
\]
Let $p_l$ by the $l^{th}$ prime number, and $p_{q-1} \leq k \leq p_q$

- $K$ primes larger than $q$ produce $(K, \lceil \log_{p_q} N \rceil)$-coherent matrix
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Number Theoretic Constructions

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### Number Theoretic Constructions

**Proof Elements**

<table>
<thead>
<tr>
<th>n ∈ [0, N)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>…</th>
</tr>
</thead>
<tbody>
<tr>
<td>n ≡ 0 mod 2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>…</td>
</tr>
<tr>
<td>n ≡ 1 mod 2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>…</td>
</tr>
<tr>
<td>n ≡ 0 mod 3</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>…</td>
</tr>
<tr>
<td>n ≡ 1 mod 3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>…</td>
</tr>
<tr>
<td>n ≡ 2 mod 3</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>…</td>
</tr>
<tr>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
</tr>
<tr>
<td>n ≡ 1 mod 5</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>…</td>
</tr>
<tr>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
</tr>
</tbody>
</table>

- Let $p_l$ by the $l^{th}$ prime number, and $p_{q-1} \leq k \leq p_q$
- $K$ primes larger than $q$ produce $(K, \lceil \log_{pq} N \rceil)$-coherent matrix
- Product with Discrete Fourier Transform Matrix is very sparse
- Number of rows of $\mathcal{M} = \text{Rows of } \mathcal{M} \mathcal{F}$ with nonzero entries =

$$
\sum_{j=0}^{K-1} p_{q+j} \leq \frac{7k^2 \cdot \lceil \log_k N \rceil^2}{\epsilon^2} \ln \left( \frac{3.05 \cdot k \cdot \lceil \log_k N \rceil}{\epsilon} \right)
$$
Proof Overview

1. Develop good binary measurement matrices, \( M \in \{0, 1\}^{m \times N} \), with both analytic and combinatorial structure ✓

2. Develop fast recovery methods for \( f : [0, 2\pi] \mapsto \mathbb{C} \) by utilizing the combinatorial structure of \( M \)

3. Map \( f : [0, 2\pi]^D \mapsto \mathbb{C} \) to a one dimensional function ✓
Example: Finding One Nonzero Entry

- $M$ is $5 \times 6$, $\vec{x}$ contains 1 nonzero entry.

\[
\begin{align*}
\equiv 0 \mod 2 & \quad \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 3.5 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
\equiv 1 \mod 2 & \quad \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix} \\
\equiv 0 \mod 3 & \quad \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \\
\equiv 1 \mod 3 & \quad \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \\
\equiv 2 \mod 3 & \quad \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}
\end{align*}
\]

- Reconstruct entry index via Chinese Remainder Theorem
- Two estimates of the entry’s value

SAVED ONE TEST!
Example: Finding One Nonzero Entry

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\begin{align*}
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\equiv 1 \mod 2 & \quad \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix} \\
\equiv 0 \mod 3 & \quad \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \\
\equiv 1 \mod 3 & \quad \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \\
\equiv 2 \mod 3 & \quad \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}
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\]

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\[
\begin{pmatrix}
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 \\
1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 \\
\end{pmatrix}
\begin{pmatrix}
0 \\
0 \\
3.5 \\
0 \\
0 \\
\end{pmatrix}
= 
\begin{pmatrix}
3.5 \\
0 \\
0 \\
0 \\
3.5 \\
\end{pmatrix}
\Leftarrow \text{Index } \equiv 0 \text{ mod } 2
\]
\[
\begin{pmatrix}
0 \\
0 \\
3.5 \\
0 \\
0 \\
\end{pmatrix}
\Leftarrow \text{Index } \equiv 2 \text{ mod } 3
\]

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Example: Finding One Nonzero Entry

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\[
\begin{pmatrix}
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 \\
1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
0 \\
0 \\
3.5 \\
0 \\
0
\end{pmatrix}
= 
\begin{pmatrix}
3.5 \\
0 \\
0 \\
0 \\
3.5
\end{pmatrix}
\quad \Leftarrow \quad \text{Index } \equiv 0 \mod 2
\]

\[
\begin{pmatrix}
0 \\
0 \\
3.5 \\
0 \\
0
\end{pmatrix}
\quad \Leftarrow \quad \text{Index } \equiv 2 \mod 3
\]

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\[
\begin{pmatrix}
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 \\
1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
0 \\
0 \\
3.5 \\
0 \\
0
\end{pmatrix}
= 
\begin{pmatrix}
3.5 \\
0 \\
0 \\
0 \\
3.5
\end{pmatrix}
\quad \Leftarrow \quad \text{Index } \equiv 0 \text{ mod } 2
\]

\[
\begin{pmatrix}
0 \\
0 \\
3.5 \\
0 \\
0
\end{pmatrix}
\quad \Leftarrow \quad \text{Index } \equiv 2 \text{ mod } 3
\]

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Example: Finding One Nonzero Entry

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\[
\begin{pmatrix}
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 \\
1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
0 \\
0 \\
3.5 \\
0 \\
0 \\
0
\end{pmatrix} =
\begin{pmatrix}
3.5 \\
0 \\
0 \\
0 \\
3.5
\end{pmatrix}
\iff \text{Index } \equiv 0 \mod 2
\]

- Index $\equiv 2 \mod 3$

- Reconstruct entry index via Chinese Remainder Theorem
- Two estimates of the entry’s value

SAVED ONE TEST!
Example: Finding One Nonzero Fourier Coefficient

\[
\begin{pmatrix}
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 \\
1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 \\
\end{pmatrix}
\begin{pmatrix}
0 \\
0 \\
3.5 \\
0 \\
0 \\
0 \\
\end{pmatrix}
= 
\begin{pmatrix}
3.5 \\
0 \\
0 \\
0 \\
0 \\
3.5 \\
\end{pmatrix}
\]

- We only utilize 4 samples
- Computed Efficiently using 2 FFTs
- Reconstruct frequency index via Chinese Remainder Theorem
- Two estimates of nonzero Fourier coefficient

SAVED TWO SAMPLES!
Example: Finding One Nonzero Fourier Coefficient

\[
\begin{pmatrix}
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 \\
1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 1
\end{pmatrix}
\cdot \mathcal{F}_{6\times6}^{-1}
\cdot
\begin{pmatrix}
0 \\
0 \\
3.5 \\
0 \\
0 \\
0
\end{pmatrix}
= 
\begin{pmatrix}
3.5 \\
0 \\
0 \\
3.5
\end{pmatrix}
\]

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Example: Finding One Nonzero Fourier Coefficient

\[
\begin{pmatrix}
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 \\
1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 1
\end{pmatrix}
\mathcal{F}_{6 \times 6}
\begin{pmatrix}
0 \\
3.5 \\
0 \\
0 \\
0 \\
3.5
\end{pmatrix}
= 
\begin{pmatrix}
3.5 \\
0 \\
0 \\
3.5
\end{pmatrix}
\]

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SAVED TWO SAMPLES!
Example: Finding One Nonzero Fourier Coefficient

\[
\begin{pmatrix}
\sqrt{\frac{3}{2}} & 0 & 0 & \sqrt{\frac{3}{2}} & 0 & 0 \\
\sqrt{\frac{3}{2}} & 0 & 0 & -\sqrt{\frac{3}{2}} & 0 & 0 \\
* & 0 & * & 0 & * & 0 \\
* & 0 & * & 0 & * & 0 \\
* & 0 & * & 0 & * & 0 \\
\end{pmatrix} \cdot \begin{pmatrix}
F_{6 \times 6}^{-1}
\end{pmatrix}
\begin{pmatrix}
0 \\
3.5 \\
0 \\
0 \\
0 \\
3.5 \\
\end{pmatrix}
= \begin{pmatrix}
3.5 \\
0 \\
0 \\
0 \\
3.5 \\
\end{pmatrix}
\]

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\[
\begin{pmatrix}
\sqrt{\frac{3}{2}} & 0 & 0 & \sqrt{\frac{3}{2}} & 0 & 0 \\
\sqrt{\frac{3}{2}} & 0 & 0 & -\sqrt{\frac{3}{2}} & 0 & 0 \\
* & 0 & * & 0 & * & 0 \\
* & 0 & * & 0 & * & 0 \\
* & 0 & * & 0 & * & 0 \\
\end{pmatrix}
\cdot
\begin{pmatrix}
\mathcal{F}_{6 \times 6}^{-1} \\
\begin{pmatrix}
0 \\
0 \\
3.5 \\
0 \\
0 \\
3.5 \\
\end{pmatrix}
\end{pmatrix}
= \begin{pmatrix}
3.5 \\
0 \\
0 \\
3.5 \\
\end{pmatrix}
\]

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SAVED TWO SAMPLES!
Example: Finding One Nonzero Fourier Coefficient

\[
\begin{pmatrix}
\sqrt{3} \cdot \psi_{2 \times 2} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0
\end{pmatrix} \\
\sqrt{2} \cdot \psi_{3 \times 3} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0
\end{pmatrix}
\end{pmatrix}
\cdot
\begin{pmatrix}
\mathcal{F}^{-1}_{6 \times 6} \begin{pmatrix} 0 & 0 \\
0 & 3.5 \\
0 & 0 \\
3.5 \\
0 & 0 \\
0 & 0
\end{pmatrix}
\end{pmatrix}
= \begin{pmatrix} 3.5 \\
0 \\
0 \\
0 \\
3.5
\end{pmatrix}
\]

- We only utilize 4 samples
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SAVED TWO SAMPLES!
Example: Finding One Nonzero Fourier Coefficient

\[
\begin{pmatrix}
\sqrt{3} \cdot \psi_{2 \times 2} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\
\end{pmatrix} \\
\sqrt{2} \cdot \psi_{3 \times 3} \cdot \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3.5 \\
\end{pmatrix} \\
\end{pmatrix} \cdot \begin{pmatrix}
\mathcal{F}_{6 \times 6}^{-1} \\
\begin{pmatrix} 0 \\ 0 \\ 3.5 \\ 0 \\ 0 \\ 0 \\
\end{pmatrix} \\
\end{pmatrix} = \begin{pmatrix} 3.5 \\ 0 \\ 0 \\ 3.5 \\
\end{pmatrix}
\]

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SAVED TWO SAMPLES!
Example: Finding One Nonzero Fourier Coefficient

\[
\begin{pmatrix}
\sqrt{3} \cdot \psi_{2 \times 2} \cdot \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0
\end{pmatrix}

& \begin{pmatrix}
\sqrt{2} \cdot \psi_{3 \times 3} \cdot \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0
\end{pmatrix}

\end{pmatrix} \cdot \begin{pmatrix}
\mathcal{F}^{-1}_{6 \times 6} \\
0 \\
3.5 \\
0 \\
0 \\
3.5
\end{pmatrix} = \begin{pmatrix}
3.5 \\
0 \\
0 \\
3.5
\end{pmatrix}
\]

- We only utilize 4 samples
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Example: Finding One Nonzero Fourier Coefficient

\[
\begin{pmatrix}
\sqrt{3} \cdot \psi_{2 \times 2} \cdot \\
\sqrt{2} \cdot \psi_{3 \times 3} \cdot \\
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
\end{pmatrix}
\begin{pmatrix}
\mathcal{F}_{6 \times 6}^{-1} \\
\end{pmatrix}
\begin{pmatrix}
0 \\
3.5 \\
0 \\
0 \\
0 \\
3.5 \\
\end{pmatrix}
= 
\begin{pmatrix}
3.5 \\
0 \\
0 \\
3.5 \\
\end{pmatrix}
\]

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SAVED TWO SAMPLES!
Proof Overview

1. Develop good binary measurement matrices, $\mathcal{M} \in \{0, 1\}^{m \times N}$, with both analytic and combinatorial structure ✓

2. Develop fast recovery methods for $f : [0, 2\pi] \mapsto \mathbb{C}$ by utilizing the combinatorial structure of $\mathcal{M}$ ✓

3. Map $f : [0, 2\pi]^D \mapsto \mathbb{C}$ to a one dimensional function ✓
Recovery results hold more generally (not just in the Fourier case)
  • Related expansions (e.g., Chebyshev by a change of variable to Fourier)
  • For arbitrary dictionaries if 'function evaluations’ replaced by 'linear measurements’

Nonuniform recovery results can be obtained using structured random matrices produced by randomly sampling \((K, \alpha)\)-coherent matrix rows

For number theoretic constructions analytic number theory provides nice results and tools for explicitly bounding required samples, etc.
Conclusion

- Recovery results hold more generally (not just in the Fourier case)
  - Related expansions (e.g., Chebyshev by a change of variable to Fourier)
  - For arbitrary dictionaries if ‘function evaluations’ replaced by ‘linear measurements’

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- For number theoretic constructions analytic number theory provides nice results and tools for explicitly bounding required samples, etc.
Recovery results hold more generally (not just in the Fourier case)
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Nonuniform recovery results can be obtained using structured random matrices produced by randomly sampling \((K, \alpha)\)-coherent matrix rows

For number theoretic constructions analytic number theory provides nice results and tools for explicitly bounding required samples, etc.
Questions?

Thank You!