Quasi Monte-Carlo Integration for Parametric and Stochastic Operator Equations

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We present regularity theory for PDEs with uncertain input parameters, related to recently developed, deterministic, high-order Quasi-Monte Carlo quadratures.

Admissible problems include (linear or semilinear) elliptic or parabolic partial differential equations with uncertain parameters, and the corresponding Bayesian inverse problems as formulated in [5].

Based on a parametrization of the distributed uncertainty, the computational problem reduces to integration problems over infinite-dimensional parameter spaces. Based on a holomorphy condition on the parameteric dependence as in [1], we present regularity estimates of the parametric integrand functions and for uniform prior measure on the parameter uncertainty in classes of weighted RKHS with hybrids of product and SPOD weights introduced in [2].

Related recent results (joint with J. Dick, F. Kuo, T. LeGhia and D. Nuyens) [3] on dimension independent convergence rates of the deterministic, higher order QMC quadrature for integrand functions in weighted function spaces will be presented in detail in the lecture of Josef Dick.

The (dimension-independent) convergence rates of the resulting QMC quadratures are found to be completely determined by the sparsity class of the uncertain distributed input parameter and are, in particular, higher than those of MCMC methods in terms of the number $M$ of solutions of the forward problems.

The density of the posterior measure in Bayesian inverse problems as considered in [5, 4] is shown to belong to the class of admissible integrand functions with a hybrid of product and SPOD weights.

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References


