Peter Koepke

Academic career

1978	Diploma, University of Bonn
1979	Master of Arts, University of California, Berkeley, CA, USA
1984	PhD, University of Freiburg
1984 - 1987	Feodor-Lynen-Fellowship, Alexander von Humboldt Foundation, and Junior Research Fellow, Wolfson College, Oxford, England, UK
1987 - 1990	Assistant Professor (C1), Habilitation, University of Freiburg
Since 1990	Professor (C3), University of Bonn

Invited Lectures

2009	Computability in Europe, Heidelberg
2009	Effective Mathematics of the Uncountable 2009, CUNY, New York, USA
2010	Set Theory, Classical and Constructive, Amsterdam, Netherlands
2010	Set Theory, Model Theory, Generalized Quantifiers and Foundations of Mathematics, Helsinki, Finland
2011	14th Congress of Logic, Methodology and Philosophy of Science, Nancy, France
2013	CUNY Logic Workshop, New York, USA
2013	Proof 2013, Bern, Switzerland
2013	Mal'cev Meeting, Novosibirsk, Russia
2014	60th birthday conference of Philip Welch, Bristol, England, UK
2015	Set Theory, Carnegie Mellon University, Pittsburgh, PA, USA
2015	Philosophy of Mathematics Seminar, Oxford, England, UK
2015	European Set Theory Conference, Cambridge, England, UK
2016	Menachem Magidor 70th Birthday Conference, The Hebrew University of Jerusalem, Israel

Research Projects and Activities

DFG project "Complexity and Definability at Higher Cardinals"

Research profile

My set theoretical research focusses around the construction and analysis of models of set theory with various combinatorial properties, using the methods of forcing, inner models, and symmetric models. My main interest is on models having strong closure properties in the form of large cardinals like measurable cardinals and canonical strengthenings. Model constructions allow to classify set theoretic properties in terms of the existence of large cardinals: A model with large cardinals is extended by forcing to a model of the combinatorial property; conversely assuming the property one defines an inner model of set theory with large cardinals.
Recent results of this type concern "small" measurable cardinals within the bounded Gitik model (with A. Apter and I. Dimitriou), model theoretic properties about the existence of elementary substructures with
cardinality constraints (with A. Apter and I. Dimitriou) or the existence of forcing extensions in which successor cardinals of the ground model become singular (with D. Adolf and A. Apter).
Sometimes large cardinals can be eliminated: we constructed models of set theory without the axiom of choice in which the generalized continuum hypothesis formulated by F. Hausdorff can be violated in rather arbitrary ways (with A. Fernengel).
The model of constructible sets by K. Gödel can be obtained in several ways. I developed the approach by ordinal computability, combining Turing computability and uncountable set theory.

Calibrating certain parameters of ordinal computability one obtains initial segments of Gödel's model of various heights. With A. Morozov I determined the segment corresponding to infinite time Blum-Shub-Smale machines.
The foundations of mathematics encompass the (natural) language of mathematics. The Naproche system developed in the logic group shows that natural language processing can be applied to mathematics: Naproche prototypically accepts proof texts in natural mathematical language and checks their correctness (with M. Cramer). For longer texts though we are experiencing a combinatorial explosion in the current setup since the background automatic theorem prover is given too many premises for its proof search.
In future research the set-theoretical models mentioned above will be analyzed further. What remains of the ground model large cardinal properties in the Gitik model? Are the strongly compact cardinals of the ground model still Rowbottom cardinals? What is the cardinal arithmetic of infinite sums and products in the model with Fernengel? How does Shelah's theory of possible cofinalities behave in that model? Can the model be modified so that the axiom of choice holds for countable families? Work on the minimality of Prikry forcing with Gitik and Kanovei which shows that all non-trivial subforcings of Prikry forcing are themselves Prikry forcings is to be finalized.
I shall use ordinal computability for the fine structural analysis of constructible sets. One can use ordinal computability theory to reconstruct an existing but cumbersome fine structure theory of J. Silver. There should however be more a direct approaches in which the typical objects of fine structure can be obtained by computations. Constructible models are also relevant in the project to generalized descriptive set theory. Such models provide wellordering of low definitional complexity, and hence counterexamples to regularity properties of low complexity.
The Naproche approach shall be extended to natural mathematical argumentation in collaborations with the Isabelle community (L. Paulson and M. Wenzel) and A. Paskevich (SAD system). We shall pursue the thesis that the combination of Naproche techniques with large-scale technical systems like Isabelle may overcome the complexity problems indicated above by intelligent premise selection. We shall apply the methods to logical and set-theoretical texts. Results of these experiments can yield philosophical insights into the nature of mathematical proofs.

Former Research Area L Algorithms in transfinite set theory:

Ordinal computability provides a unifying spectrum of computabilities, parameterized by ordinal time and space bounds, where the computable sets correspond to Borel, Δ^1_2-, Gödel-constructible sets and other classes. We prove fundamental properties of these classes via computability. A fine structure theory for the constructible universe can be defined using ordinal algorithms. Other processes like dynamic systems or Blum-Shub-Smale computations will be continued into the infinite ordinals.
Formal mathematics:
In the Naproche project (Natural language proof checking), we connect ordinary mathematical texts with fully formal mathematics. The efficient use of automatic theorem provers for proof checking depends on the right choice of proof obligation sent to the prover. We study selection and preprocessing algorithms based on heuristics and natural language triggers. We are reformulating Landau's Grundlagen der Analysis into human readable and computer checked formats.
We shall combine Naproche techniques with established powerful formal mathematics systems.
Infinitary combinatorics:
We shall examine cardinal arithmetic for singular cardinals without assuming the axiom of choice, expanding on joint work with Apter and Gitik. We conjecture that infinitary cardinal exponentiation can take arbitrary cardinal values as long as some basic monotonicity is respected. This contrasts with the singular cardinal behaviour if the axiom of choice is assumed.

Supervised theses

Master theses: 1
Diplom theses: 55, currently 5
PhD theses: 12, currently 5

Selected PhD students

Ralf Schindler (1996): “The Core Model up to one Strong Cardinal”,
now Professor (C4), Mathematics, University of Münster
Merlin Carl (2011): “Alternative finestructural and computational approaches to constructibility”,
now Assistant Professor, Mathematics, University of Konstanz
Marcos Cramer (2013): “Proof-checking mathematical texts in controlled natural language”,
now Research Assistant, Computer Science, University of Luxembourg
Benjamin Seyfferth (2013): “Three models of ordinal computability”,
now Coordinator of Studies, Mathematics, University of Darmstadt

Habilitations

Heike Mildenberger (1998), now Professor, University of Freiburg
Benedikt Löwe (2005), now Professor, University of Amsterdam, Netherlands, and University of Hamburg

Selected publications

[1] Arthur W. Apter, Ioanna M. Dimitriou, and Peter Koepke. All uncountable cardinals in the gitik model are almost ramsey and carry rowbottom filters. *MLQ Math. Log. Q.*, 62(3):225–231, 2016.
[2] Arthur W. Apter, Ioanna M. Dimitriou, and Peter Koepke. The first measurable cardinal can be the first uncountable regular cardinal at any successor height. *MLQ Math. Log. Q.*, 60(6):471–486, 2014.
[3] Peter Koepke and Julian J. Schlöder. The gödel completeness theorem for uncountable languages. *Formalized Mathematics*, 20:199–203, 2012.
[4] Moti Gitik and Peter Koepke. Violating the singular cardinals hypothesis without large cardinals. *Israel J. Math.*, 191(2):901–922, 2012.
[5] P. Koepke and P. D. Welch. Global square and mutual stationarity at the $\aleph_n$. *Ann. Pure Appl. Logic*, 162(10):787–806, 2011.
[6] Peter Koepke. Turing computations on ordinals. *Bulletin of Symbolic Logic*, 11(3):377–397, 2005.
[7] Peter Koepke. Extenders, embedding normal forms, and the martin-steel-theorem. *J. Symbolic Logic*, 63(3):1137–1176, 1998.
[8] Sy D. Friedman and Peter Koepke. An elementary approach to the fine structure of l. *Bull. Symbolic Logic*, 3(4):453–468, 1997.