Sparse recovery and compressed sensing in inverse problems

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(joint work with Evelyn Herrholz)

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June 7, 2010
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Motivation

Sparse recovery and compressed sensing in inverse problems

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Motivation
References


■ Solve $Ax = y$ or $F(x) = y$

($A, F : X \to Y, X, Y$ Hilbert spaces)
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Often noisy data $\|y^\delta - y\| \leq \delta$
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■ The inverse problem is ill-posed
Motivation

- Solve $Ax = y$ or $F(x) = y$

($A, F : X \rightarrow Y, X, Y$ Hilbert spaces)

- Often noisy data $\|y^\delta - y\| \leq \delta$

- The inverse problem is ill-posed

- Often: incomplete data or no data sampled at adequate rate
Solve $Ax = y$ or $F(x) = y$

($A, F : X \to Y$, $X, Y$ Hilbert spaces)

Often noisy data $\|y^\delta - y\| \leq \delta$

The inverse problem is ill-posed

Often: incomplete data or no data sampled at adequate rate

Some a-priori knowledge required:

solution has sparse representation / is compressible
Sparse recovery for inverse problems
Tikhonov approach for linear/nonlinear problems:

\[
\min_x \| F(x) - y^\delta \|^2 + 2\alpha \| x \|^p
\]
Tikhonov approach for linear/nonlinear problems:

$$\min_x \| F(x) - y^\delta \|^2 + 2\alpha \| x \|^p$$

Linear case: [Daubechies, Defrise, DeMol 2004]

$$x^{n+1} = S_\alpha (x^n + \gamma A^*(y - Ax^n))$$

Nonlinear case: [Ramlau, T. 2005, 2007]

Many alternatives and improvements (incomplete list ...)
- steepest descent
- domain decomposition
- semi-smooth Newton methods
- projection methods
  [Bredies, Daubechies, Fornasier, Lorenz, Loris, ...]
- adaptive iteration
  [Ramlau, T., Zhariy and Dahlke, Fornasier, Raasch, ...]
Tikhonov approach for linear/nonlinear problems:

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Linear case: [Daubechies, Defrise, DeMol 2004]

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\[
x^{n+1} = S_\alpha(x^n + \gamma F'(x^{n+1})^*(y - F(x^n)))
\]
Tikhonov approach for linear/nonlinear problems:

$$\min_x \| F(x) - y^\delta \|_2^2 + 2\alpha \| x \|_p$$

Linear case: [Daubechies, Defrise, DeMol 2004]

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Many alternatives and improvements (incomplete list ...)

- steepest descent, domain decomposition, semi-smooth Newton methods, projection methods [Bredies, Daubechies, Fornasier, Lorenz, Loris, ...]
- adaptive iteration [Ramlau, T., Zhariy and Dahlke, Fornasier, Raasch, ...]
Sparsity through projection + acceleration

Problem:

\[ x^{n+1} = S_\alpha(x^n + \gamma A^*(y - Ax^n)) \]

overshooting (dynamics discussed by Daubechies, Fornasier, Loris 2008)
Sparse recovery and compressed sensing in inverse problems

Sparse recovery for inverse problems

Sparsity through projection + acceleration

**Problem:**

\[ x^{n+1} = S_\alpha(x^n + \gamma A^*(y - Ax^n)) \]

**Overshooting** (dynamics discussed by Daubechies, Fornasier, Loris 2008)

**Alternative:**

(Daubechies et.al. 2008, Borries, T. 2009 (nonlinear case))

\[
\min_{x \in B_R} \left\{ \|Ax - y\|^2 \right\} \quad \text{where} \quad B_R := \{ x \in \ell_2; \|x\|_1 \leq R \}
\]

\[ x^{n+1} = P_R(x^n + \gamma^n A^*(y - Ax^n)) \]
Vector-valued setup and joint sparsity formulation:

\[
\min_{d \in C_{R}^{p,q}} \|y^{\delta} - Ax\|^2
\]

where

\[
C_{R}^{p,q} = \left\{ x \in (\ell_2(\Lambda))^m : \Psi_{p,q}(x) := \sum_{\ell=1}^{m} \left( \sum_{\lambda \in \Lambda} |x_{\ell,\lambda}|^p \right)^{\frac{q}{p}} \leq R \right\}
\]
Vector-valued setup and joint sparsity formulation:

$$\min_{d \in C_{R}^{p,q}} \| y^{\delta} - Ax \|^2$$

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Special case $p = 2, q = 1$:

$$P_{R}(x) = (S_{\mu}(\{x_{1,\lambda}\}_{\lambda \in \Lambda}), \ldots, S_{\mu}(\{x_{m,\lambda}\}_{\lambda \in \Lambda}))$$
Vector-valued setup and joint sparsity formulation:

$$\min_{d \in C_{R}^{p,q}} \| y^{\delta} - Ax \|^{2}$$

where

$$C_{R}^{p,q} = \left\{ x \in (\ell_{2}(\Lambda))^{m} : \Psi_{p,q}(x) := \sum_{\ell=1}^{m} \left( \sum_{\lambda \in \Lambda} |x_{\ell,\lambda}|^{p} \right)^{\frac{q}{p}} \leq R \right\}$$

Special case $p = 2, q = 1$:

$$P_{R}(x) = (S_{\mu}(\{x_{1,\lambda}\}_{\lambda \in \Lambda}), \ldots, S_{\mu}(\{x_{m,\lambda}\}_{\lambda \in \Lambda}))$$

with

$$S_{\mu}(\{x_{\ell,\lambda}\}_{\lambda \in \Lambda}) = \frac{\{x_{\ell,\lambda}\}_{\lambda \in \Lambda}}{\| \{x_{\ell,\lambda}\}_{\lambda \in \Lambda} \|_{\ell_{2}(\Lambda)}} \max(\| \{x_{\ell,\lambda}\}_{\lambda \in \Lambda} \|_{\ell_{2}(\Lambda)} - \mu, 0)$$
Compressively sensed data
Compressed Sensing Idea:

\[ \tilde{A} \times x = \tilde{d} \]

Suppose there is a signal \( x \).
Compressed Sensing Idea:

\[
\begin{bmatrix}
y \\
x
\end{bmatrix} = \begin{bmatrix} \tilde{A} \end{bmatrix}
\]

Suppose we measure just a few linear samples. Can we reconstruct \( x \)?
Compressed Sensing Idea:

Yes! If $x$ is sparse in a given “uncoherent” basis or frame, i.e.

$$x = B d$$
Compressed Sensing Idea:

\[ y = \tilde{A} x \]

Yes! If \( x \) is sparse in a given “uncoherent” basis or frame, i.e.

\[ x = B d \]

\( \tilde{A} \) is \( p \times m \) matrix with \( p \ll m \)
Compressed Sensing Idea:

\[ y = \tilde{A} \]

Yes! If \( x \) is sparse in a given “uncoherent” basis or frame, i.e.

\[ x = B d \]

- \( \tilde{A} \) is \( p \times m \) matrix with \( p \ll m \)
- \( d \) is \( k \)-sparse, then \( p \geq 2k \) data points are required (stability: \( p \geq ck \log(m/p) \))
Compressed Sensing Idea:

Yes! If $x$ is sparse in a given "uncoherent" basis or frame, i.e.

$$x = B d$$

- $\tilde{A}$ is $p \times m$ matrix with $p \ll m$
- $d$ $k$-sparse, then $p \geq 2k$ data points are required (stability: $p \geq ck \log(m/p)$)
- $A = \tilde{A}B$ has to fulfill a **restricted isometry property**
Definition (restricted isometry property~ RIP)

For each integer $k = 1, 2, \ldots$, define the isometry constant $\delta_k$ of a sensing matrix $A$ as the smallest number such that

\[(1 - \delta_k)\|d\|_{\ell_2}^2 \leq \|Ad\|_{\ell_2}^2 \leq (1 + \delta_k)\|d\|_{\ell_2}^2\]

holds for all $k$-sparse vectors $d$. A vector is said to be $k$-sparse if it has at most $k$ non-vanishing entries.
Optimization problem:

\[
\min_{d} \|d\|_{\ell_1} \quad \text{subject to} \quad y = Ad \quad \text{or} \quad \|y^\delta - Ad\| \leq \delta \quad (\star)
\]
Optimization problem:

\[
\min_{d} \|d\|_{\ell_1} \quad \text{subject to } y = Ad \text{ or } \|y^{\delta} - Ad\| \leq \delta \quad (*)
\]

(E. Candes 2008)

Let A satisfy RIP with \(\delta_{2k} < \sqrt{2} - 1\) and \(d^k\) denote the best \(k\)-term approximation, then the solution \(d^*\) to (*) obeys

\[
\|d^* - d\|_{\ell_2} \leq C_0 k^{-1/2} \|d^k - d\|_{\ell_1}
\]

or in case of noisy data \(y^{\delta}\) with \(\|y^{\delta} - y\|_{\ell_2} \leq \tilde{\delta}\)

\[
\|d^* - d\|_{\ell_2} \leq C_0 k^{-1/2} \|d^k - d\|_{\ell_1} + C_1 \delta
\]
Ill-posed sensing model

and sparse recovery
Indirect measurement $Kx$:

$Kx = \tilde{A}y$

Consequences

$K$ may be ill-conditioned $\sim$ RIP is usually lost.
Indirect measurement $Kx$:

$$\begin{bmatrix} y \\ \tilde{A} \end{bmatrix} = \begin{bmatrix} K \\ B \end{bmatrix}$$

**Consequences**

$K$ may be ill-conditioned $\Rightarrow$ RIP is usually lost.

**Question:** How to overcome this problem and how to adjust the finite-dimensional CS setting to inverse problems?
Sparse recovery and compressed sensing in inverse problems

Ill-posed sensing model and sparse recovery

(setup by Y. Eldar, 2009)

Let $X$ be a Hilbert space and $X_m \subset X$ the reconstruction space

$$X_m = \left\{ x \in X, \ x = \sum_{\ell=1}^{m} \sum_{\lambda \in \Lambda} d_{\ell,\lambda} a_{\ell,\lambda}, \ d \in (\ell_2 (\Lambda))^m \right\}$$
Let $X$ be a Hilbert space and $X_m \subset X$ the reconstruction space

\[
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\]

**Sparse structure:** support set $\mathcal{I} \subset \{1, \ldots, m\}$

\[
X_k = \left\{ x \in X, \ x = \sum_{\ell \in \mathcal{I}, |\mathcal{I}|=k} \sum_{\lambda \in \Lambda} d_{\ell,\lambda} a_{\ell,\lambda}, \ d \in (\ell_2(\Lambda))^m \right\}
\]
Analysis and synthesis operators

\[ T_\phi : X \to \ell_2 \text{ via } x \mapsto x = \left\{ \langle x, \phi_\lambda \rangle \right\}_{\lambda \in \Lambda} \]

\[ T_\phi^* : \ell_2 \to X \text{ via } x \mapsto \sum_{\lambda \in \Lambda} x_\lambda \phi_\lambda. \]
representation of solution: $x = T_a^*d$
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data model: $y = T_vKT_a^*d$
representation of solution:  \( x = T_a^* d \)

data model:  \( y = T_v K T_a^* d \)

compressed sampling:

\[
\begin{pmatrix}
  s_1, \lambda \\
  \vdots \\
  s_p, \lambda \\
\end{pmatrix}
= A \begin{pmatrix}
  v_1, \lambda \\
  \vdots \\
  v_m, \lambda \\
\end{pmatrix}
\text{ for all } \lambda \in \Lambda
\]
representation of solution: $x = T_a^* d$

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\end{pmatrix} \quad \text{for all } \lambda \in \Lambda$$

compressed data model: $y = T_s K T_a^* d$
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Assume

\[
\langle v_\ell, \lambda, K a_{\ell'}, \lambda' \rangle = \kappa_{\ell, \lambda} \delta_{\ell, \ell'} \delta_{\lambda, \lambda'}
\]
compressed data model: \( y = T_s KT_a^* d \)

Assume
\[
\langle v_\ell, \lambda, Ka_{\ell'}, \lambda' \rangle = \kappa_{\ell, \lambda} \delta_{\ell, \ell'} \delta_{\lambda, \lambda'}
\]

then for all \( \lambda \in \Lambda \):
\[
y_\lambda = AD_\lambda d_\lambda \quad \text{with} \quad D_\lambda = diag(\kappa_{1, \lambda}, \ldots, \kappa_{m, \lambda})
\]
compressed data model: $y = T_s K T^*_a d = ADd$

Assume

$$\langle v_\ell, \lambda, K a_{\ell'}, \lambda' \rangle = \kappa_{\ell, \lambda} \delta_{\ell, \ell'} \delta_{\lambda, \lambda'}$$

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then for all $\lambda \in \Lambda$:

$$y_\lambda = AD_\lambda d_\lambda \text{ with } D_\lambda = diag(\kappa_{1, \lambda}, \ldots, \kappa_{m, \lambda})$$

Problem: $AD$ does not satisfy RIP
For all $\lambda \in \Lambda$ consider optimization problem

$$\min_{d_\lambda \in B_R} \|y^\delta_\lambda - AD_\lambda d_\lambda\|_2^2 + \alpha \|d_\lambda\|_2^2,$$

where $B_R = \{d_\lambda \in \ell_2 : \|d_\lambda\|_1 \leq R\}$.
For all $\lambda \in \Lambda$ consider optimization problem

$$\min_{d\lambda \in B_R} \|y_\lambda - AD\lambda d\lambda\|_2^2 + \alpha \|d\lambda\|_2^2,$$

where $B_R = \{d\lambda \in \ell_2 : \|d\lambda\|_1 \leq R\}$

Corresponding model operator $L^2 := D\lambda A^* AD\lambda + \alpha I$, leading to stabilized RIP

$$(\kappa_{min}^2 (1 - \delta_k) + \alpha)\|x\|^2 \leq \|Lx\|^2 \leq (\kappa_{max}^2 (1 + \delta_k) + \alpha)\|x\|^2$$
Theorem (Herrholz, T. 2010)

Define $d^{\dagger}_\lambda$ as the $B_R$-best approximation to the true solution. Assume $R$ was chosen such that the solution $d_\lambda \not\in B_R$ and that

$$0 \leq \delta_{2k} < \frac{(1 + \sqrt{2})\kappa_{\min}^2 - \kappa_{\max}^2 + \sqrt{2}\alpha}{(1 + \sqrt{2})\kappa_{\min}^2 + \kappa_{\max}^2}.$$

Then the minimizer $d^*_\lambda$ satisfies

$$\|d^*_\lambda - d_\lambda\|_2 \leq C_0 k^{-1/2} \|d^k_\lambda - d_\lambda\|_1 + C_1 \delta + C_2 \|L(d^{\dagger}_\lambda - d_\lambda)\|_2 + C_3 \sqrt{\alpha} R.$$
Treatment of all $\lambda$ simultaneously by a joint sparsity formulation:

$$\min_{d \in C_{R}^{2,1}} \|y^{\delta} - ADd\|_{(\ell_{2}(\Lambda))}^{2} + \alpha \|d\|_{(\ell_{2}(\Lambda))}^{m}$$
Treatment of all $\lambda$ simultaneously by a joint sparsity formulation:

$$\min_{d \in C^2_1} \|y^\delta - ADd\|_2^2 + \alpha \|d\|_2^2$$

where

$$C^2_1 = \{d \in (\ell_2(\Lambda))^m : \Psi_{2,1}(d) \leq R\}$$
Theorem (Herrholz, T. 2010)

Let $d^\dagger$ be the $B_R$-best approximation to the true solution, $R$ as before and

$$0 \leq \delta_{2k} < \frac{(1 + \sqrt{2})\kappa_{min}^2 - \kappa_{max}^2 + \sqrt{2}\alpha}{(1 + \sqrt{2})\kappa_{min}^2 + \kappa_{max}^2}.$$

Then the minimizer $d^*$ satisfies

$$\|d^* - d\|_{(\ell_2(\Lambda))^m} \leq C_0 k^{-1/2} \Psi_{2,1}(d^k - d) + C_1 \delta$$

$$+ C_2 \|L(d^\dagger - d)\|_{(\ell_2(\Lambda))^m} + C_3 \sqrt{\alpha} R.$$
Discussion on the balancing on $\delta_{2k}$ of $\alpha$:
Discussion on the balancing on \( \delta_{2k} \) of \( \alpha \):

- Given \( K \) and sensing matrix \( A \):

\[
\frac{(1 + \delta_{2k})\kappa^2_{\text{max}} - (1 + \sqrt{2})(1 - \delta_{2k})\kappa^2_{\text{min}}}{\sqrt{2}} < \alpha
\]
Discussion on the balancing on $\delta_{2k}$ of $\alpha$:

Given $K$ and sensing matrix $A$:

$$
\frac{(1 + \delta_{2k})\kappa_{\text{max}}^2 - (1 + \sqrt{2})(1 - \delta_{2k})\kappa_{\text{min}}^2}{\sqrt{2}} < \alpha
$$

If $\delta_{2k} < 1$, a stabilization becomes necessary if

$$
\frac{1 + \delta_{2k}}{1 - \delta_{2k}} \cdot \frac{\kappa_{\text{max}}^2}{\kappa_{\text{min}}^2} > 1 + \sqrt{2}.
$$
Discussion on the balancing on $\delta_{2k}$ of $\alpha$:

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If $\delta_{2k} \geq 1$, lower bound is always positive.
Discussion on the balancing on $\delta_{2k}$ of $\alpha$:

- Given $K$ and sensing matrix $A$:

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\frac{(1 + \delta_{2k})\kappa_{\text{max}}^2 - (1 + \sqrt{2})(1 - \delta_{2k})\kappa_{\text{min}}^2}{\sqrt{2}} < \alpha
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If $\delta_{2k} < 1$, a stabilization becomes necessary if

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\frac{1 + \delta_{2k}}{1 - \delta_{2k}} \cdot \frac{\kappa_{\text{max}}^2}{\kappa_{\text{min}}^2} > 1 + \sqrt{2}.
$$

If $\delta_{2k} \geq 1$, lower bound is always positive.

- Specializing the case $\delta_{2k} < 1$ to $\kappa_{\text{max}}^2 = \kappa_{\text{min}}^2 = 1$:

$$
\delta_{2k} > \sqrt{2} - 1
$$
Given $K$, stabilize first and suppose $A$ can be chosen accordingly:

$$\frac{\kappa_{max}^2 - (1 + \sqrt{2})\kappa_{min}^2}{\sqrt{2}} < \alpha.$$
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$$\frac{\kappa_{\text{max}}^2 - (1 + \sqrt{2})\kappa_{\text{min}}^2}{\sqrt{2}} < \alpha.$$ 

No stabilization necessary ($\alpha = 0$):

$$1 \leq \frac{\kappa_{\text{max}}^2}{\kappa_{\text{min}}^2} < 1 + \sqrt{2}$$
Given $K$, stabilize first and suppose $A$ can be chosen accordingly:

$$\frac{\kappa_{\text{max}}^2 - (1 + \sqrt{2})\kappa_{\text{min}}^2}{\sqrt{2}} < \alpha.$$ 

No stabilization necessary ($\alpha = 0$):

$$1 \leq \frac{\kappa_{\text{max}}^2}{\kappa_{\text{min}}^2} < 1 + \sqrt{2}$$

$$\delta_{2k} < \frac{1 + \sqrt{2} - \frac{\kappa_{\text{max}}^2}{\kappa_{\text{min}}^2}}{1 + \sqrt{2} + \frac{\kappa_{\text{max}}^2}{\kappa_{\text{min}}^2}}$$
Given $K$, stabilize first and suppose $A$ can be chosen accordingly:

$$\frac{\kappa^2_{max} - (1 + \sqrt{2})\kappa^2_{min}}{\sqrt{2}} < \alpha.$$  

No stabilization necessary ($\alpha = 0$):

$$1 \leq \frac{\kappa^2_{max}}{\kappa^2_{min}} < 1 + \sqrt{2}$$

$$\delta_{2k} < \frac{1 + \sqrt{2} - \frac{\kappa^2_{max}}{\kappa^2_{min}}}{1 + \sqrt{2} + \frac{\kappa^2_{max}}{\kappa^2_{min}}}$$

Stabilization necessary ($\alpha > 0$):

$$1 + \sqrt{2} \leq \frac{\kappa^2_{max}}{\kappa^2_{min}}$$
Given $K$, stabilize first and suppose $A$ can be chosen accordingly:

$$\frac{\kappa_{\text{max}}^2 - (1 + \sqrt{2})\kappa_{\text{min}}^2}{\sqrt{2}} < \alpha.$$ 

No stabilization necessary ($\alpha = 0$):

$$1 \leq \frac{\kappa_{\text{max}}^2}{\kappa_{\text{min}}^2} < 1 + \sqrt{2}$$

$$\delta_{2k} < \frac{1 + \sqrt{2} - \kappa_{\text{max}}^2/\kappa_{\text{min}}^2}{1 + \sqrt{2} + \kappa_{\text{max}}^2/\kappa_{\text{min}}^2}$$

Stabilization necessary ($\alpha > 0$):

$$1 + \sqrt{2} \leq \kappa_{\text{max}}^2/\kappa_{\text{min}}^2$$

$$\delta_{2k} < \frac{1 + \sqrt{2} - \kappa_{\text{max}}^2/\kappa_{\text{min}}^2 + \sqrt{2}\alpha/\kappa_{\text{min}}^2}{1 + \sqrt{2} + \kappa_{\text{max}}^2/\kappa_{\text{min}}^2}$$
Numerical experiment

(MATLAB)
Numerical experiment

- \( a_{\ell, \lambda} - \text{sinc- wavelets, } \ell \in \{-3, -2, \ldots, 4, 5\}, \text{ i.e. } m = 9 \)
- \( K \) convolution operator \( \rightarrow D_\lambda \)
- \( x \) is 2-sparse,

\[
\begin{align*}
d_{1,21} &= 3, & d_{4,20} &= -1, & d_{4,25} &= 2
\end{align*}
\]

- \( y_\lambda^\delta = AD_\lambda d_\lambda + \delta \)
$K$ corresponds to diagonal entries $\kappa_{\ell, \lambda}$:
minimization of

\[
\min_{d \in C_{R}^{2,1}} \|y^\delta - ADd\|_{(\ell_2(\Lambda))^p}^2 + \alpha \|d\|_{(\ell_2(\Lambda))^m}^2
\]

yields:
Algorithmic aspects:
Summary:

- **Algorithms for sparse recovery:**
  - projected steepest descent, step length control, vector-valued, joint sparsity

- **Suggestion for a setup for incomplete/compressed data:**
  - semi-infinite dimensional

- **In the compressed data model:**
  - balance between the sensing properties of $A$ and stabilization by $\alpha$
  - choice of $\alpha$ restricted (typically $\alpha \not\to 0$ to ensure CS recovery)

- **Future work:**
  - apply infinite CS model (A. Hansen, 2010)
  - combine compressed data model with adaptive operator evaluation
Workshop on CS, Sparsity and Inverse Problems

September 6-7, 2010 at TU Braunschweig, Germany

http://www.dfg-spp1324.de/nuhagtools/event/make.php?event=cssip10