The Complexity of Numerical Integration of Smooth Functions

We study the complexity of multivariate integration for a number of classes of smooth functions. We consider deterministic algorithms using function values.

In particular, we show that the curse of dimensionality holds for the class of \( r \) times continuously differentiable \( d \)-variate functions whose values are at most one the curse holds iff the bound on all derivatives up to order \( r \) does not go to zero faster than \( d^{-1/2} \). We also consider the case of infinitely many differentiable functions and prove the curse if the bounds on the successive derivatives are appropriately large. The proof technique is based on a volume estimate of a neighborhood of the convex hull of \( n \) points which decays exponentially quickly if \( n \) is small relative to \( d \).

For \( r = \infty \), we also study conditions for quasi-polynomial, weak and uniform weak tractability. We prove that the Clenshaw Curtis Smolyak algorithm leads to weak tractability of the problem of integration of \( d \)-variate analytic functions defined on the unit cube with directional derivatives of all orders bounded by 1. This seems to be the first positive tractability result for the Smolyak algorithm for a normalized and unweighted problem. The space of integrands is not a tensor product space and therefore we have to develop a different proof technique. We use the polynomial exactness of the algorithm as well as an explicit bound on the operator norm of the algorithm.

This is joint work with Erich Novak, Henryk Wozniakowski and Mario Ullrich.