Weighted Hilbert Spaces of Functions of Infinitely Many Variables: Embeddings and Integration

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We study some issues that arise for integration problems for functions of infinite many variables, which have recently been studied intensively in the literature.

The setting is based on a reproducing kernel $k$ for functions on a domain $D$, a family of non-negative weights $\gamma_u$, where $u$ varies over all finite subsets of $\mathbb{N}$, and a probability measure $\rho$ on $D$. For the construction of the function space we consider the tensor product kernels $k_u(x,y) = \prod_{j \in u} k(x_j, y_j)$ with $x, y \in D^u$, as well as the weighted superposition $K = \sum_u \gamma_u k_u$.

We show that, under mild assumptions, $K$ is a reproducing kernel on a properly chosen domain $X \subseteq D^{\mathbb{N}}$, and $H(K)$ is the orthogonal sum of the spaces $H(\gamma_u k_u)$. Thereafter, we relate two approaches to define an integral for functions on $H(K)$, namely via a canonical representer or with respect to the product measure $\rho^{\mathbb{N}}$ on $D^{\mathbb{N}}$. In particular, we provide sufficient conditions for the two approaches to lead to the same notion of integral. Finally, we study embeddings between weighted Hilbert spaces in the particular case of product weights, i.e., $\gamma_u = \prod_{j \in u} \gamma_j$ for a sequence of positive reals $\gamma_j$.

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