JOINT SESSIONS

Workshops in Analytic Number Theory and Real Analysis
Supported by the Hausdorff Center for Mathematics and
Hausdorff Research Institute for Mathematics
Wegelerstr. 10 ∘ 16 – 17 July 2014

Wednesday July 16 ∘ 14:00
Efficient congruencing and a Diophantine inequality of Bourgain and Demeter
Trevor Wooley (University of Bristol)
As a consequence of recent work concerning the proof of the ℓ^2 decoupling conjecture, Bourgain and Demeter show that for each fixed $k \geq 2$ and $C > 0$, the Diophantine system

$$|n_1^k + n_2^k + n_3^k - n_4^k - n_5^k - n_6^k| \leq CN^{k-2}$$
$$n_1 + n_2 + n_3 - n_4 - n_5 - n_6 = 0$$

has $O(N^{3+\epsilon})$ integral solutions with $n_i \leq N$. We explore the consequences of the efficient congruencing method (from Vinogradov's mean value theorem) for this problem and its generalisations.

Wednesday July 16 ∘ 15:00
Bob Hough's solution of Erdős's covering congruences conjecture
Ben Green (University of Oxford)
One of Paul Erdős's very favourite questions (possibly his favourite of all) was the following: Let M_0 be arbitrary. Can you cover $\mathbb{Z}$ with finitely many congruence conditions $a \pmod m$, $m > M_0$, at most one for each m? In 2013, Bob Hough showed that the answer is no if M_0 is sufficiently large. That is, there is some constant C such that in any covering of $\mathbb{Z}$ by finitely many congruences to distinct moduli m, at least one of the m's must be at most C. I will present his proof.

Wednesday July 16 ∘ 17:00
A Sharpened Hausdorff-Young Inequality
Michael Christ (UC Berkeley)
One of the most fundamental facts about the Fourier transform is the Hausdorff-Young inequality, which states that for any locally compact Abelian group, the Fourier transform maps L^p boundedly to L^q, where the two exponents are conjugate and $p \in [1, 2]$. For Euclidean space, the optimal constant in this inequality was found by Babenko for q an even integer, and by Beckner for general exponents. Lieb showed that all extremizers are Gaussian functions. This is a uniqueness theorem; these Gaussians form the orbit of a single function under the group of symmetries of the inequality.

We establish a stabler form of uniqueness for $1 < p < 2$: (i) If a function f nearly achieves the optimal constant in the inequality, then f must be close in norm to a Gaussian. (ii) There is a quantitative bound involving the square of the distance to the nearest Gaussian. The qualitative form (i) can be equivalently formulated as a precompactness theorem in the style of the calculus of variations. Form (ii) is a strengthening of the inequality. Ingredients taken from additive combinatorics are at the heart of the analysis. Arithmetic progressions, of arbitrarily high rank, play an important part.

Thursday July 17 ◦ 14:00
Local-Global in Thin Orbits and Applications
Alex Kontorovich (Yale University)
We will discuss an ongoing program with Jean Bourgain to study local-global phenomena in orbits that are "thin." Consequences include applications to numerical integration, pseudorandom sequences, and Diophantine geometry.

Thursday July 17 ◦ 15:00
Equidistribution estimates for the primes
Terence Tao (UC Los Angeles)
One of the basic questions in analytic number theory is to understand how the prime numbers are distributed in arithmetic progressions; this information can be combined with sieve-theoretic tools to obtain results such as the recent establishment of an infinite sequence of bounded gaps between the prime numbers. For progressions of small modulus, one can obtain satisfactory results using the theory of Dirichlet L-functions, but for progressions of large modulus, even the generalised Riemann hypothesis is insufficient to obtain useful distributional results for all progressions. However, a celebrated and very useful theorem of Bombieri and Vinogradov unconditionally gives equidistribution of arithmetic progressions *on the average*, as long as the spacing of the progression is less than the square root of the magnitude of the entries.

It has been a major challenge to break this "square root barrier" and obtain stronger equidistribution estimates on the primes. Limited results in this direction were initially obtained by Bombieri, Fouvry, Friedlander, and Iwaniec, but last year there was a significant advance by Yitang Zhang, who obtained a robust family of such estimates, by combining the dispersion method of Linnik with known estimates on exponential sums. These estimates have since been strengthened, with somewhat simplified proofs, by the online collaborative Polymath project, and have been used to improve the bounds on gaps between primes. In this talk, we will survey these equidistribution estimates, and give some indication of their proofs.

Thursday July 17 ◦ 17:00
Small gaps between primes
James Maynard (Montreal)
It is believed that there should be infinitely many pairs of primes which differ by 2; this is the famous twin prime conjecture. More generally, it is believed that for every positive integer m there should be infinitely many sets of m primes, with each set contained in an interval of size roughly $m \log m$. Although proving these conjectures seems to be beyond our current techniques, recent progress has enabled us to obtain some partial results. We will introduce a refinement of the 'GPY sieve method' for studying these problems. This refinement will allow us to show (amongst other things) that $\liminf_n (p_{n+m} - p_n) < \infty$ for any integer m, and so there are infinitely many bounded length intervals containing m primes. We also discuss some extensions of this result.