Lattice points in cubature and collocation

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Lattice rules for numerical integration were introduced by Korobov [2]. They were constructed to achieve the optimal rate of convergence for numerical integration of functions expressed by a Fourier series with coefficients decaying according to a hyperbolic cross. To control the exponential dependency on the number of dimensions, Sloan and Woźniakowski [4] introduced weighted function spaces. Optimal lattice rules in weighted spaces can be constructed using the fast component-by-component algorithm [3]. Recently also functions expressed in cosine series were studied [1] for numerical integration.

Spectral collocation and reconstruction methods using Fourier expansions have been studied before in combination with lattice points. In a current manuscript [5], together with Suryanarayana and Cools, we investigate the use of lattice points for the approximation and collocation of $d$-variate non-periodic functions with frequency support on a hyperbolic cross of cosine series. We show that rank-1 lattice points can be used as collocation points in the approximation of non-periodic functions and these lattice points can be constructed by a component-by-component algorithm.

References


