Abstract

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Let $Q_M = M + (0, 1)^n$, $M \in \mathbb{Z}^n$, be unit cubes in $\mathbb{R}^n$ and let $\Gamma = \{x^j\}_{j=1}^k \subset \mathbb{R}^n$. We discuss the behaviour of the discrepancy function

$$\text{disc}_{\Gamma, A}(x) = \prod_{l=1}^n (x_l - M_l) - \sum_{j: R^j_i \subset Q_M} a_j \chi_{R^j_i}(x), \quad x \in Q_M,$$

where $A = \{a_j\}_{j=1}^k \subset \mathbb{C}$, and $R^j_i$ are rectangles in $Q_M$ with $M + \bar{T}$ as upper right corner and $x^j \in Q_M$ as lower left corner, in weighted Sobolev-Besov spaces with dominating mixed smoothness. Dualizing the outcome in the sense of a related Hlawka-Zaremba identity one can say something about numerical integration

$$\sup \left| \int_{\mathbb{R}^n} f(x) \, dx - \sum_{j=1}^k a_j f(x^j) \right|$$

where the supremum is taken over unit balls in suitable weighted spaces with dominating mixed smoothness.