**Points of uncountable score**

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This talk will describe successful applications of descriptive set theory to symbolic dynamics, and list related open problems which might be similarly susceptible.

Let $\mathcal{X}$ be a Polish space with metric $d$, $f : \mathcal{X} \longrightarrow \mathcal{X}$ a continuous function and $x$, $y$ points in $\mathcal{X}$. We say that $x$ *attacks* $y$, in symbols, $x \curvearrowright_f y$, if

$$\forall \varepsilon > 0 \, \forall m {:}{\in} \mathbb{N} \, \exists \ell \, [\ell \geqslant m \;\&\; d(f^\ell(x), y) < \varepsilon].$$

Our notation relates to one familiar to dynamicists: $x \curvearrowright_f y$ iff $y \in \omega_f(x)$.

The *score*, $\theta(a, f)$, of a point $a$ in $\mathcal{X}$ with respect to the function $f$, is defined to be the least ordinal $\theta$ such that $A^\theta(a, f) = A^{\theta+1}(a, f)$, where we define recursively a shrinking sequence of sets by $A^0(a, f) = \omega_f(a)$, $A^{\nu+1}(a, f) = \{x \mid \exists y(y \in A^\nu(a, f) \;\&\; y \curvearrowright_f x)\}$ and for a limit ordinal $\lambda$, $A^\lambda(a, f) = \bigcap_{\nu<\lambda} A^\nu(a, f)$.

Proposition [3] *For any $\mathcal{X}$, $f$ and $a$ as above, $\theta(a, f) \leqslant \omega_1$.*

Example *Baire space*, the space of infinite sequences of natural numbers, often denoted by $\mathcal{N}$ or ${}^\omega\omega$, which for each finite such sequence $r$ has the basic open set $\{\alpha \mid \alpha \restriction \ell h(r) = r\}$. The (backward) *shift function* $\mathfrak{s} : \mathcal{N} \longrightarrow \mathcal{N}$ is given by $\mathfrak{s}(\alpha)(n) = \alpha(n + 1)$.

Proposition [3] *In Baire space there are points of $\mathfrak{s}$-score any given countable ordinal.*

Theorem [4] *There is a recursive point in Baire space with $\mathfrak{s}$-score $\omega_1$.*

An unpublished transfer theorem of Christian Delhommé shows that a point of uncountable score and points of all countable scores will exist in Cantor space ${}^\omega 2$.

[3] A. R. D. Mathias, Delays, recurrence and ordinals, *Proc. London Math. Soc. (3)* **82** (2001) 257–298; MR 2001j:03087.

[4] A. R. D. Mathias, Analytic sets under attack, *Math. Proc Cam. Phil. Soc* **138** (2005) 465–485,