Workshop

Discrepancy, Numerical Integration and Hyperbolic Cross Approximation

Hausdorff Center for Mathematics
University of Bonn

September 23-27, 2013

Organizers
Tino Ullrich (University of Bonn)
Vladimir N. Temlyakov (University of South Carolina)

Discrepancy, Numerical Integration and Hyperbolic Cross Approximation

Many multi-parameter real-world problems are modeled on spaces of multivariate functions in d variables, where d may be very large. Since these problems can almost never be solved analytically, one is interested in suitable model assumptions and good approximate solutions within a reasonable computing time.

Already more than fifty years ago classes of functions with bounded mixed derivative had been used in numerical integration and approximation for the first time. In this context numerical integration dealt with two powerful methods, Quasi Monte Carlo and Sparse Grids. Approximation methods used polynomials with frequencies from hyperbolic crosses. It was understood later that classes of functions with bounded mixed derivative appear naturally in discrepancy, the small ball problem from probability theory and in problems involving Wiener sheet measure. Furthermore, it has been shown recently that the wave functions of the electronic Schrödinger equation possess a bounded mixed derivative. As a consequence, the by now classical model of function spaces with dominating mixed smoothness attracted more and more interest among researchers from different fields.

So far, the theory of hyperbolic cross approximation and its applications has developed into a beautiful practically useful theory which still has a number of important open problems to work on. In fact, the basic ideas in approximating and integrating such functions have been mainly developed in the former Soviet Union during the last fifty years. However, many remarkable and highly important theoretical results in this context remained unrecognized in the western scientific community.

Since the Institute for Numerical Simulation in Bonn is one of the main centers for the design of Sparse Grid methods in scientific computing, an exchange of ideas is highly appreciated. The workshop aims at bringing together experts from different communities to learn new approaches and techniques from each other. In particular, we aim at identifying possible new research directions and methods that may arise from the combination of different expertises.

Program Overview

Time	Monday	Tuesday	Wednesday	Thursday	Friday
08:30 - 08:50	Opening				
08:50 - 09:40	Triebel	Yserentant	Woźniakowski	Prestin	Ritter
09:40 - 10:30	Skriganov	Đinh	Wasilkowski	Kunis	Gnewuch
10:30 - 11:10	Coffee break				
11:10 - 12:00	Bilyk	Sickel	Nuyens	Schneider	Heinrich
12:00 - 14:20	Lunch break		Excursion	Lunch break	
14:20 - 15:10	Novak	Schwab		Oswald	
15:10 - 16:00	Hinrichs	Dick		Delvos	
16:00 - 16:30	Coffee break			Coffee break	
16:30 - 17:20	Temlyakov	Harbrecht		Discussion	
⋮					
20:00 -			Conference Dinner		

Some Information

Location

All lectures of the workshop will take place in the Lipschitz Lecture Hall at the Mathematik-Zentrum, Endenicher Allee 60, first floor (European counting).

Getting there

Please, have a look at the map in the middle of this booklet. Probably the most convenient way to get to the workshop venue is by walking. From Bonn Central Station you can take one of the bus lines 604, 605, 606, 607 to bus stop Kaufmannstraße — or simply a taxi.

Registration on Monday

You find the reception desk in front of Lipschitz Lecture Hall. It opens at 8am on Monday morning.

Contact Information

Tino Ullrich	+4917682132816	tino.ullrich@hcm.uni-bonn.de
Vladimir Temlyakov		temlyakovusc@gmail.com
Sebastian Mayer	+491733456846	mayer@ins.uni-bon.de
Glenn Byrenheid	+491743044555	glenn.byrenheid@hcm.uni-bonn.de

Lunch

There are several options for lunch. The most convenient way is to take your lunch at the local university cafeteria, which is located on the opposite side of the Mathematik-Zentrum. There is no special conference offer, you just pay the usual price for guests. This means you will get a meal for 3–5 Euro.

Alternatively, you find a couple of restaurants at 10–15 minutes walking distance. The average meal price will be around 8–15 Euro.

Dinner on Wednesday

On Wednesday evening we have planned a dinner at the restaurant "Em Höttche" in the city center of Bonn. It is a traditional German restaurant and "typical" for this region. Unfortunately, our budget

rules do not allow to cover the dinner via the workshop budget. So all participants will have to cover their own expenses.

We arranged a selection of dishes from which you can choose. You can find the special menu on the last page of this booklet. Please let us know when registering on Monday whether you wish to participate. We would appreciate if you could choose your preferred dish in advance. Please tell us your choice at the reception desk or, alternatively, write an email to Sebastian Mayer until Monday evening.

Em Höttche
Markt 4
53111 Bonn.
http://www.em-hoettche.de

Excursion on Wednesday

You can choose between two excursions. Sign-up will be possible at the registration desk or via mail to Sebastian Mayer until Monday evening. Both excursions start around 1:15pm in front of the Mathematik-Zentrum.

(A) Trip to Cologne with guided tour through Cologne Cathedral. We take a train to Cologne (30 km) where we have a 1h guided tour through Cologne Cathedral, Northern Europe's largest church building. Afterwards we pass over Rhine river via Hohenzollern Bridge which is known for all the locks which couples leave there to prove their ties. On the other side of the Rhine river we then visit the KölnTriangle building to have a wonderful view over the city of Cologne.

The expenses for this excursion will be around 25 Euro. The exact price depends on the number of participants, which is limited to 25 persons.

(B) Boat trip on the Rhine river south to Königswinter and walk to castle ruin "Drachenfels". We have a 50 minutes boat trip on the Rhine river to Königswinter, which is south of Bonn. There we walk up a steep path to the castle ruin Drachenfels (3km), where you can enjoy the beautiful panorama of Rhine river and the mountains "Siebengebirge". For those who mind walking there is also a cog railway (Zahnradbahn) up the hill. After having walked down to Königswinter we return by tram to Bonn Central Station.

The price for boat and train tickets is around 15 Euro.

WLAN

You should receive a voucher with login details at the registration desk. In case you don't have access details yet or you face any problems, please contact Sebastian Mayer.

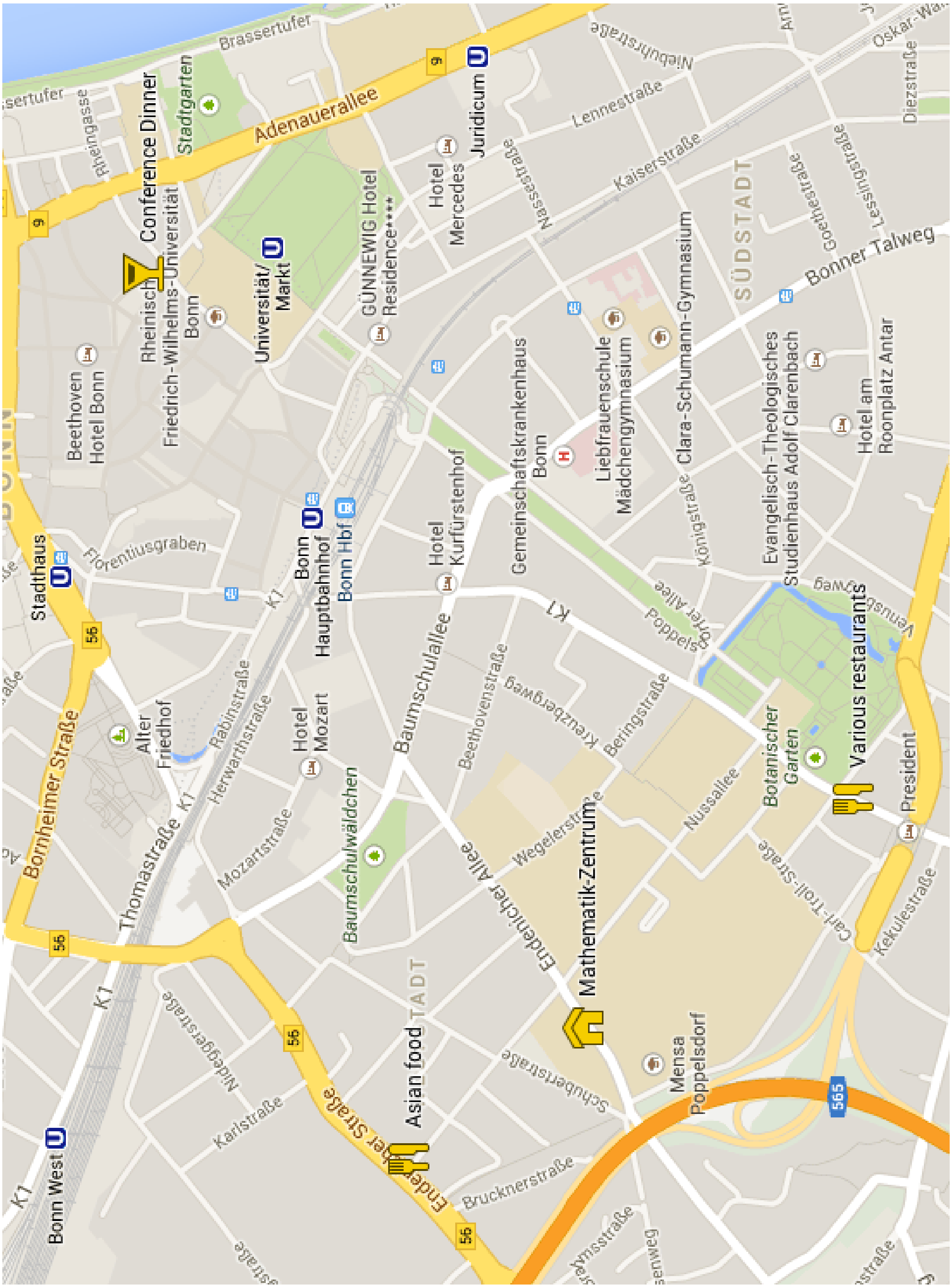
Library

Please feel free to use the mathematics library in the Mathematik-Zentrum (Endenicher Allee 60). It contains a large collection of books and journals. The entrance to the library is on the ground floor.

Acknowledgements

The organizers would like to thank the Hausdorff Center for Mathematics for the generous financial support, which made this workshop possible. In addition, they also acknowledge the support by the German Research Foundation (DFG) within the Emmy-Noether program and their kind assistance in various problems concerning Visa applications.

Special thanks go to Laura Siklossy for the great support in all administrative issues, especially Hotel booking, and to Gunder-Lily Sievert and her team for organizing the coffee breaks. We further acknowledge the efforts by Glenn Byrenheid and Sebastian Mayer for guiding the excursions, helping with practical issues during the workshop and, last but not least, for producing this booklet.


Conference Dinner
Mathematik-Zentrum
Asian food
Various restaurants
Adenauerallee
Stadtgarten
Rheinisch Friedrich-Wilhelms-Universität Bonn
Universität/Markt
GÜNNEWIG Hotel Residence*****
Hotel Mercedes
Juridicum
Beethoven Hotel Bonn
Stadthaus
Florentiusgraben
Bonn Hauptbahnhof
Bonn Hbf
Hotel Kurfürstenhof
Gemeinschaftskrankenhaus Bonn
Liebfrauenschule Mädchengymnasium
Clara-Schumann-Gymnasium
SÜDSTADT
Evangelisch-Theologisches Studienhaus Adolf Clarenbach
Hotel am Roonplatz Antar
Bonner Talweg
Kaiserstraße
Lennestraße
Baumschulallee
Hotel Mozart
Alter Friedhof
Bornheimer Straße
Thomastraße
Baumschulwäldchen
Endenicher Allee
Wegelerstraße
Nussallee
Botanischer Garten
President
Poppelsdorfer Allee
Mensa Poppelsdorf
Endenicher Straße
Bonn West
Karlstraße
Brucknerstraße
Schubertstraße
Carl-Troll-Straße
Kekuléstraße
Brassertufer

Program

Monday, September 23

8.30 - 8.50 Opening

8:50 - 9:40 Hans Triebel
Weighted discrepancy and numerical integration in function spaces

9:40 - 10:30 Maxim M. Skriganov
On mean values of the L_q-discrepancies of point distributions

10:30 - 11:10 *Coffee Break*

11:10 - 12:00 Dmitriy Bilyk
Discrepancy estimates, small ball inequalities, and hyperbolic cross approximations

12:00 - 14:20 *Lunch*

14:20 - 15:10 Erich Novak
Complexity of oscillatory integrals

15:10 - 16:00 Aicke Hinrichs
The Complexity of numerical integration of smooth functions

16:00 - 16:30 *Coffee break*

16:30 - 17:20 Vladimir N. Temlyakov
Open problems

Tuesday, September 24

8:50 - 9:40 Harry Yserentant
Mixed derivatives and hyperbolic cross spaces in quantum mechanics

9:40 - 10:30 Đinh Dũng
Hyperbolic crosses in high-dimensional approximations

10:30 - 11:10 *Coffee break*

11:10 - 12:00 Winfried Sickel
Approximation numbers of Sobolev embeddings

12:00 - 14:20 *Lunch*

14:20 - 15:10 Christoph Schwab
Quasi Monte-Carlo integration for parametric and stochastic operator equations

15:10 - 16.00 Josef Dick
Higher order Quasi Monte-Carlo methods for PDEs with random coefficients

16:00 - 16:30 *Coffee break*

16:30 - 17:20 *Helmut Harbrecht*
On multilevel quadrature for elliptic stochastic partial differential equations

Wednesday, September 25

8:50 - 9:40 Henryk Woźniakowski
New notions of tractability for analytic multivariate problems

9:40 - 10:30 Grzegorz W. Wasilkowski
Tractability of approximation of ∞-variate functions with bounded mixed partial derivatives

10:30 - 11:10 *Coffee break*

11:10 - 12:00 Dirk Nuyens
Lattice points in cubature and collocation

12:00 - 20:00 *Excursion*

20:00 - *Conference Dinner*

Thursday, September 26

8:50 - 9:40 Jürgen Prestin
Periodic shift-invariant spaces

9:40 - 10:30 Stefan Kunis
Low rank approximations and fast algorithms

10:30 - 11:10 *Coffee break*

11:10 - 12:00 Reinhold Schneider
Hierarchical tensor representation and best bilinear approximation

12:00 - 14:20 *Lunch*

14:20 - 15:10 Peter Oswald
On preconditioners for sparse grid discretizations

15:10 - 16:00 Franz-Jürgen Delvos
Blended Fejér-type approximation

16:00 - 16:30 *Coffee break*

16:30 - 17:20 *Discussion*

Friday, September 27

8:50 - 9:40 Klaus Ritter
Weighted Hilbert spaces of functions of infinitely many variables: embeddings and integration

9:40 - 10:30 Michael Gnewuch
Optimal randomized algorithms for integration on function spaces with underlying ANOVA decomposition

10:30 - 11:10 *Coffee break*

11:10 - 12:00 Stefan Heinrich
Randomized complexity of parametric problems

Abstracts

Monday, 8.50 - 9.40

Weighted discrepancy and numerical integration in function spaces

Hans Triebel
University of Jena

Let $Q_M = M + (0,1)^n$, $M \in \mathbb{Z}^n$, be unit cubes in $\mathbb{R}^n$ and let $\Gamma = \{x^j\}_{j=1}^k \subset \mathbb{R}^n$. We discuss the behaviour of the discrepancy function

$$\mathrm{disc}_{\Gamma,A}(x) = \prod_{l=1}^{n}(x_l - M_l) - \sum_{j:R_\Gamma^j \subset Q_M} a_j \, \chi_{R_\Gamma^j}(x), \qquad x \in Q_M,$$

where $A = \{a_j\}_{j=1}^k \subset \mathbb{C}$, and R_Γ^j are rectangles in Q_M with $M + \overline{1}$ as upper right corner and $x^j \in Q_M$ as lower left corner, in weighted Sobolev-Besov spaces with dominating mixed smoothness. Dualizing the outcome in the sense of a related Hlawka-Zaremba identity one can say something about numerical integration

$$\sup \Big| \int_{\mathbb{R}^n} f(x)\, dx - \sum_{j=1}^{k} a_j \, f(x^j) \Big|$$

where the supremum is taken over unit balls in suitable weighted spaces with dominating mixed smoothness.

Monday, 9.40 - 10.30

On mean values of the L_q-discrepancies of point distributions

Maxim M. Skriganov
Steklov Institute of Mathematics of the Russian Academy of Sciences

It will be shown in the talk that a large number of results on uniform distributions can be given as direct corollaries of two theorems on mean values of the L_q-discrepancies. As a corollary of these two theorems it can be shown that the best possible orders of the L_q-discrepancies of point distributions can be attained on very small averaging subsets. Furthermore, with special conditions these averaging subsets may even collapse to a single point. Such specific distributions can be called as self-averaging.

Monday, 11.10 - 12.00

Discrepancy estimates, small ball inequalities, and hyperbolic cross approximations

Dmitriy Bilyk
University of Minnesota

In his pioneering work on irregularities of distribution, K. Roth came up with the idea that the essential information about the behavior of discrepancy function can be extracted by projecting this function onto the span of Haar functions supported by dyadic rectangles with fixed volume (i.e. with fixed product of frequencies). This observation, which closely resembles the concept of hyperbolic cross approximation in approximation theory, has been used in almost every important advance on discrepancy function estimates. Similar ideas have been used in other subjects and problems, such as the entropy estimates for mixed derivative function spaces or small deviations estimates for the multiparameter Gaussian processes. These problems and conjectures seem to revolve around the so-called "small ball inequality" – an analytic estimate involving linear combinations of wavelets with frequencies living in a hyperbolic cross.

While the sharp two-dimensional versions of these problems have been resolved (in particular by methods coming from the theory of one-dimensional lacunary Fourier series), higher-dimensional analogues are still elusive and remain poorly understood, sometimes even on the level of conjectures. In this talk we shall survey the main ideas, connections, and methods in this circle of questions, and will explain some classical as well as recent results in this area.

Monday, 14.20 - 15.10

Complexity of oscillatory integrals

Erich Novak
University of Jena

For $s \in \mathbb{N}$ let

$$H^s = \{f : [0,1] \to \mathbb{C} \mid f^{(s-1)} \text{ is abs. cont., } f^{(s)} \in L_2\}$$

with

$$\langle f, g \rangle_s = \sum_{\ell=0}^{s} \left\langle f^{(\ell)}, g^{(\ell)} \right\rangle_0,$$

where $\langle f, g\rangle_0 = \int_0^1 f(x)\,\overline{g(x)}\,\mathrm{d}x$. What is the complexity of the approximate computation of oscillatory integrals of the form

$$S_k(f) = \int_0^1 f(x)\,\mathrm{e}^{-2\pi\,i\,kx}\,\mathrm{d}x,$$

where $k \in \mathbb{Z}$ and $f \in H^s$? The worst case error of A_n is

$$e(A_n) = \sup_{f \in H^s,\, \|f\|_s \le 1} |S_k(f) - A_n(f)|,$$

the nth minimal worst case error is

$$e(n, k, s) := \inf_{A_n} e(A_n).$$

We improve the upper bounds by Domínguez, Graham and Smyshlyaev (2011) and prove matching lower bounds. For the initial error we obtain for $k \neq 0$

$$e(0, k, s) = \frac{\beta_k}{2\pi|k|} \quad \text{with} \quad \beta_k \in [0.96\ldots, 1.41\ldots].$$

Main result:

$$\frac{c_s}{(n + |k|)^s} \;\le\; e(n, k, s) \;\le\; \frac{C_s}{(n + |k| - 2s + 1)^s},$$

for all $k \in \mathbb{Z}$ and $n \ge 2s$.

Monday, 15.10 - 16.00

The complexity of numerical integration of smooth functions

Aicke Hinrichs
University of Rostock

We study the complexity of multivariate integration for a number of classes of smooth functions. We consider deterministic algorithms using function values.

In particular, we show that the curse of dimensionality holds for the class of r times continuously differentiable d-variate functions whose values are at most one the curse holds iff the bound on all derivatives up to order r does not go to zero faster than $d^{-1/2}$. We also consider the case of infinitely many differentiable functions and prove the curse if the bounds on the successive derivatives are appropriately large. The proof technique is based on a volume estimate of a neighborhood

of the convex hull of n points which decays exponentially quickly if n is small relative to d.

For $r = \infty$, we also study conditions for quasi-polynomial, weak and uniform weak tractability. We prove that the Clenshaw Curtis Smolyak algorithm leads to weak tractability of the problem of integration of d-variate analytic functions defined on the unit cube with directional derivatives of all orders bounded by 1. This seems to be the first positive tractability result for the Smolyak algorithm for a normalized and unweighted problem. The space of integrands is not a tensor product space and therefore we have to develop a different proof technique. We use the polynomial exactness of the algorithm as well as an explicit bound on the operator norm of the algorithm.

This is joint work with Erich Novak, Henryk Woźniakowski and Mario Ullrich.

Monday, 16.30 - 17.20

Open problems

Vladimir N. Temlyakov
University of South Carolina

Open problems in discrepancy, numerical integration and hyperbolic cross approximation.

Tuesday, 8.50 - 9.40

Mixed derivatives and hyperbolic cross spaces in quantum mechanics

Harry Yserentant
Technische Universität Berlin

The eigenfunctions of electronic Schrödinger operators and their exponentially weighted counterparts possess, roughly speaking, mixed weak derivatives of fractional order ϑ for $\vartheta < 3/4$ in the Sobolev space H^1. The bound $3/4$ is best possible and can neither be reached nor surpassed. Such results are important for the study of sparse grid-like expansions of the wave functions and show that their asymptotic convergence rate measured in terms of the number of ansatz functions

involved does not deteriorate with the number of electrons. The resulting rate of convergence (not to speak of the actually reachable speed of convergence) is, however, low. The question arises which other properties of the wavefunctions, and what kind of nonlinear approximation methods as well, could help to improve this situation.

Tuesday, 9.40 - 10.30

Hyperbolic crosses in high-dimensional approximations

Đinh Dũng
Vietnam National University, Hanoi

We will discuss new results on explicit-in-dimension estimates of the integer points in hyperbolic crosses and applications in high-dimensional approximations, in particular, the high-dimensional problems of n-width and ε-dimension (or more general, information complexity) and its tractability for some classes of functions having mixed smoothness.

This talk is based on the recent joint works with Tino Ullrich and Alexey Chernov from the Hausdorff Center for Mathematics, Bonn.

References

[1] Dinh Dũng and Tino Ullrich, N-Widths and ε-dimensions for high dimensional approximations, *Foundations Comp. Math.* (2013), DOI 10.1007/s10208-013-9149-9.

[2] Alexey Chernov and Dinh Dũng, New estimates for the cardinality of high-dimensional hyperbolic crosses and approximations of functions having mixed smoothness, Manuscript.

Tuesday, 11.10 - 12.00

Approximation numbers of Sobolev embeddings

Winfried Sickel
University of Jena

First we investigate optimal linear approximations (approximation numbers) in the context of isotropic periodic Sobolev spaces $H^s(\mathbb{T}^d)$ of fractional smoothness $s > 0$ for various equivalent norms including the natural one. The error is always measured in $L_2(\mathbb{T}^d)$. Particular

emphasis is given to the dependence of all constants on the dimension d. We capture the exact decay rate in n and the exact decay order of the constants with respect to d, which is in fact polynomial.
Secondly, we consider the approximation numbers with respect to the pairs $H^s_{mix}(\mathbb{T}^d)$ and $L_2(\mathbb{T}^d)$. Here $H^s_{mix}(\mathbb{T}^d)$ denotes the Sobolev space of dominating mixed smoothness of order s. Again we are interested in the dependence of these numbers on n and d.
This is joint work with Thomas Kühn (Leipzig) and Tino Ullrich (Bonn).

Tuesday, 14.20 - 15.10

Quasi Monte-Carlo integration for parametric and stochastic operator equations

Christoph Schwab
SAM, ETH Zürich, ETH Zentrum, CH 8092 Zürich, Switzerland

We present regularity theory for PDEs with uncertain input parameters, related to recently developed, deterministic, high-order Quasi-Monte Carlo quadratures.

Admissible problems include (linear or semilinear) elliptic or parabolic partial differential equations with uncertain parameters, and the corresponding Bayesian inverse problems as formulated in [5].

Based on a parametrization of the distributed uncertainty, the computational problem reduces to integration problems over infinite-dimensional parameter spaces. Based on a holomorphy condition on the parameteric dependence as in [1], we present regularity estimates of the parametric integrand functions and for uniform prior measure on the parameter uncertainty in classes of weighted RKHS with hybrids of product and SPOD weights introduced in [2].

Related recent results (joint with J. Dick, F. Kuo, T. LeGhia and D. Nuyens) [3] on dimension independent convergence rates of the deterministic, higher order QMC quadrature for integrand functions in weighted function spaces will be presented in detail in the lecture of Josef Dick.

The (dimension-independent) convergence rates of the resulting QMC quadratures are found to be completely determined by the sparsity class of the uncertain distributed input parameter and are, in particular, higher than those of MCMC methods in terms of the number M of solutions of the forward problems.

The density of the posterior measure in Bayesian inverse problems as considered in [5, 4] is shown to belong to the class of admissible integrand functions with a hybrid of product and SPOD weights.

This research has been supported in part under ERC AdG 247277.

Tuesday, 15.10 - 16.00

Higher order Quasi Monte-Carlo methods for PDEs with random coefficients

Josef Dick
School of Mathematics and Statistics, The University of New South Wales, Sydney, Australia

PDEs with random coefficients yield challenging infinite dimensional integration problems. The integrands considered are often very smooth, however, the convergence rate is limit by the behaviour of the integrand with respect to the dimension.

In a recent paper (joint with F. Kuo, Q. T. Le Gia, D. Nuyens and Ch. Schwab) [3] we showed dimension independent convergence rates for higher order QMC quadrature for integrand functions in weighted spaces. The natural weights in this context are a generalization of POD weights previously studied by Kuo, Schwab and Sloan [2]. This method can be applied to parametric operator equations with uncertain parameters, which will be discussed in a related talk by Ch. Schwab.

References

[1] A. Cohen, A. Chkifa and Ch. Schwab, Breaking the curse of dimensionality in sparse polynomial approximation of parametric PDEs. Report 2013-25, Seminar for Applied Mathematics, ETH Zürich

[2] F. Y. Kuo, Ch. Schwab, and I. H. Sloan, Quasi-Monte Carlo finite element methods for a class of elliptic partial differential equations with random coefficients. SIAM J. Numer. Anal., 50, 3351–3374, 2012.

[3] J. Dick, Frances Y. Kuo, Quoc T. Le Gia, D. Nuyens, and Ch. Schwab, Higher order QMC Galerkin discretization for parametric operator equations, Preprint 2013 (in preparation).

[4] Cl. Schillings and Ch. Schwab, *Sparse, adaptive Smolyak algorithms for Bayesian inverse problems*, Report 2012-37, SAM, ETH Zürich. Inverse Problems (2013), (in press).

[5] Ch. Schwab and A. M. Stuart, Sparse deterministic approximation of Bayesian inverse problems, Inverse Problems **28**(4) *http* : *//dx.doi.org*/10.1088/0266 − 5611/28/4/045003, (2012).

Tuesday, 16.30 - 17.20

On multilevel quadrature for elliptic stochastic partial differential equations

Helmut Harbrecht
University of Basel

In this talk, we show that the multilevel Monte Carlo method for elliptic stochastic partial differential equations can be interpreted as a sparse grid approximation. By using this interpretation, the method can straightforwardly be generalized to any given quadrature rule for high dimensional integrals like the quasi Monte Carlo method or a Gaussian quadrature. Besides the multilevel quadrature for approximating the solution's expectation, a simple and efficient modification of the approach is proposed to compute the stochastic solution's variance. Numerical results are provided to demonstrate and quantify the approach

Wednesday, 8.50 - 9.40

New notions of tractability for analytic multivariate problems

Henryk Woźniakowski
Columbia University

For analytic multivariate problems it is reasonable to expect an exponential convergence. This implies that an ε-approximation can be computed with cost proportional to $1 + \log \varepsilon^{-1}$. For such problems, it seems reasonable to define tractability of a d-variate problem in terms of $1 + \log \varepsilon^{-1}$ and d, instead of ε^{-1} and d as it is done for non-analytic problems. In my talk I will survey recent results for this type of tractability for multivariate integration and approximation. The talk will be based on joint work with Josef Dick, Peter Kritzer, Gerhard Larcher and Friedrich Pillichshammer.

Wednesday, 9.40 - 10.30

Tractability of approximation of ∞-variate functions with bounded mixed partial derivatives

Grzegorz W. Wasilkowski
University of Kentucky

We present recent results on the tractability of ψ-weighted L_s approximation for γ-weighted Banach spaces of ∞-variate functions whose mixed partial derivatives of order r are bounded in a ω-weighted L_p norm. Functions from such spaces have a natural decomposition $f = \sum_{\mathfrak{u}} f_{\mathfrak{u}}$, where the summation is with respect to finite subsets $\mathfrak{u} \subset \mathbb{N}_+$ and each $f_{\mathfrak{u}}$ depends only on variables listed in $\mathfrak{u}$. We present corresponding *multivariate decomposition methods* and show that they lead to polynomial tractability under suitable assumptions concerning γ weights and the probability density functions ψ and ω. For instance, suppose that the cost of evaluating functions with d variables is at most exponential in d and the weights γ decay to zero sufficiently quickly. Then the cost of approximating such functions with the weighted L_s-error at most ε is proportional to $\varepsilon^{-1/(r+\min(1/s-1/p,0))}$ ignoring logarithmic terms. This is a nearly-optimal results, since (once again ignoring logarithmic terms) it equals the complexity of the same approximation problem in the univariate case.

Wednesday, 11.10 - 12.00
Lattice points in cubature and collocation

Dirk Nuyens
Department of Computer Science, KU Leuven, Belgium

Lattice rules for numerical integration were introduced by Korobov [2]. They were constructed to achieve the optimal rate of convergence for numerical integration of functions expressed by a Fourier series with coefficients decaying according to a hyperbolic cross. To control the exponential dependency on the number of dimensions, Sloan and Woźniakowski [4] introduced weighted function spaces. Optimal lattice rules in weighted spaces can be constructed using the fast component-by-component algorithm [3]. Recently also functions expressed in cosine series were studied [1] for numerical integration.

Spectral collocation and reconstruction methods using Fourier expansions have been studied before in combination with lattice points. In a current manuscript [5], together with Suryanarayana and Cools, we investigate the use of lattice points for the approximation and collocation of d-variate non-periodic functions with frequency support on a hyperbolic cross of cosine series. We show that rank-1 lattice points can be used as collocation points in the approximation of non-

periodic functions and these lattice points can be constructed by a component-by-component algorithm.

References

[1] J. Dick, D. Nuyens, and F. Pillichshammer. Lattice rules for nonperiodic smooth integrands. *Numer. Math.*, 1–33, in press, 2013.

[2] N. M. Korobov. The approximate computation of multiple integrals / Approximate evaluation of repeated integrals. *Dokl. Akad. Nauk SSSR*, 124:1207–1210, 1959. In Russian.

[3] D. Nuyens and R. Cools. Fast algorithms for component-by-component construction of rank-1 lattice rules in shift-invariant reproducing kernel Hilbert spaces. *Math. Comp.*, 75(254):903–920, 2006.

[4] I. H. Sloan and H. Woźniakowski. When are quasi-Monte Carlo algorithms efficient for high dimensional integrals? *J. Complexity*, 14(1):1–33, 1998.

[5] G. Suryanarayana, D. Nuyens, and R. Cools. Reconstruction and collocation of a class of non-periodic functions by sampling along tent-transformed rank-1 lattices. In preparation, 2013.

Thursday, 8.50 - 9.40

Periodic shift-invariant spaces

Jürgen Prestin
University of Lübeck

In this talk one-dimensional shift-invariant spaces of periodic functions are generalized to multivariate shift-invariant spaces on non-tensor product patterns. Here, we discuss matrix shifts, where M is a $(d \times d)$-integer matrix with $\det M > 1$. Accordingly, an element of the shift-invariant space spanned by a function φ is given as the linear combination

$$\sum_{\ell \in \Gamma} c_\ell \varphi(\cdot - 2\pi M^{-1}\ell),$$

where Γ denotes the full collection of coset representatives of $\mathbb{Z}^d/\Gamma\mathbb{Z}^d$. Decompositions of shift-invariant spaces are given by divisibility considerations.

For these spaces we discuss the dimension and construct interpolatory and orthonormal bases. Possible patterns are classified. The results are applied to construct multivariate orthogonal kernels of Dirichlet and de la Vallée Poussin type and the respective wavelets.

This is joint work with R. Bergmann (Kaiserslautern) and D. Langemann (Braunschweig).

Thursday, 9.40 - 10.30

Low rank approximations and fast algorithms

Stefan Kunis
University of Osnabrueck

Structural properties of specific problems are exploited by classical fast algorithms to achieve their favourable computational complexity. While more general problems would not allow for a fast algorithm in infinite precision, they often do when considered for finite accuracy. We discuss the idea of such schemes which relies on low rank approximations under certain admissibility conditions and dedicated divide and conquer strategies. Examples are given for specific discretised integral operators including so-called hierarchical matrices for asymptotically smooth kernel functions, a fast Laplace transform, and generalized fast Fourier transforms.

Thursday, 11.10 - 12.00

Hierarchical tensor representation and best bilinear approximation

Reinhold Schneider
Technische Universität Berlin

Hierarchical Tucker tensor format (Hackbusch) and Tensor Trains (TT) (Tyrtyshnikov) have been introduced recently offering stable and robust approximation by a low order cost. In case $\mathcal{V} = \bigotimes_{i=1}^{d} \mathcal{V}_i$ which is proportional to d and polynomial in the ranks. We investigate the convergence of this approach with respect to the correspondent ranks and with the total amount of data required to represent the approximate tensor. To provide examples, we refer established regularity classes, and consider (periodic) mixed Sobolev spaces and used results from Telmyakov. It turns out, that in this setting, we obtain optimal ranks, and optimal convergence w.r.t. the ranks, however our tensor representation is not optimal, so far. This result is neither surprising nor disappointing, see e.g. results of Harbrecht& Griebel, due

to the great flexibility and high adaptivity of low rank approximations. We consider also the numerical solution of parabolic PDEs in the present tensor framework constraint by the restriction to tensors of prescribed multi-linear ranks **r** and discussed best approximation rates. Due to the multi-linearity, we establish a system of low dimensional nonlinear differential equations by applying Dirac Frenkel principle, which leads to a nonlinear non-linear Galerkin framework.

Thursday, 14.20 - 15.10
On preconditioners for sparse grid discretizations

Peter Oswald
Jacobs University, Bremen

The talk discusses some old and new results on optimal preconditioning of discretizations of elliptic problems by generalized sparse-grid discritizations. The focus is on subspace correction methods with simple subspace solvers and the optimal scaling of the subproblems, to boost performance. This is joint work with M. Griebel and A. Hullmann (Bonn).

Thursday, 15.10 - 16.00
Blended Fejér-type approximation

Franz-Jürgen Delvos
Universität Siegen

Babuška introduced the concept of periodic Hilbert spaces in studying optimal approximation of linear functionals. These spaces were used to study the approximation properties of trigonometric interpolation and periodic spline interpolation. We will continue the investigation of approximation by generalized Fourier partial sums constructed by Boolean methods and consider the construction of bivariate periodic Hilbert spaces and bivariate Fejér operators. In particular, we study the approximation order of blended Fejér operators.

Friday, 8.50 - 9.40

Weighted Hilbert spaces of functions of infinitely many variables: embeddings and integration

Klaus Ritter
TU Kaiserslautern

We study some issues that arise for integration problems for functions of infinite many variables, which have recently been studied intensively in the literature.

The setting is based on a reproducing kernel k for functions on a domain D, a family of non-negative weights γ_u, where u varies over all finite subsets of $\mathbb{N}$, and a probability measure ρ on D. For the construction of the function space we consider the tensor product kernels $k_u(\mathbf{x},\mathbf{y}) = \prod_{j\in u} k(x_j, y_j)$ with $\mathbf{x},\mathbf{y} \in D^u$, as well as the weighted superposition $K = \sum_u \gamma_u k_u$.

We show that, under mild assumptions, K is a reproducing kernel on a properly chosen domain $X \subseteq D^{\mathbb{N}}$, and $H(K)$ is the orthogonal sum of the spaces $H(\gamma_u k_u)$. Thereafter, we relate two approaches to define an integral for functions on $H(K)$, namely via a canonical representer or with respect to the product measure $\rho^{\mathbb{N}}$ on $D^{\mathbb{N}}$. In particular, we provide sufficient conditions for the two approaches to lead to the same notion of integral. Finally, we study embeddings between weighted Hilbert spaces in the particular case of product weights, i.e., $\gamma_u = \prod_{j\in u} \gamma_j$ for a sequence of positive reals γ_j.

Joint work with Michael Gnewuch (TU Kaiserslautern), Mario Hefter (TU Kaiserslautern), and Sebastian Mayer (U Bonn). Partially supported by the DFG within Priority Program 1324 and by the Center for Mathematical and Computational Modelling $(CM)^2$.

Friday, 9.40 - 10.30

Optimal randomized algorithms for integration on function spaces with underlying ANOVA decomposition

Michael Gnewuch
University of Kiel

In this talk we present upper and lower error bounds for the infinite-dimensional numerical integration problem on weighted Hilbert spaces with norms induced by an underlying ANOVA decomposition. Here

the weights model the relative importance of different groups of variables. We have results for randomized algorithms in two different cost models and our error bounds are in both settings sharp in the case of product weights and finite-intersection weights. The constructive upper error bounds are based on randomized multilevel altorithms in the first cost model and randomized changing dimension algorithms in the second cost model.

As example spaces of integrands we discuss Sobolev spaces with different degrees of smoothness. In this setting we use quasi-Monte Carlo multilevel and changing dimension algorithms based on (interlaced) scrambled polynomial lattice rules. These algorithm can obtain higher order convergence rates that are arbitrarily close to the optimal convergence rate.

The talk is based on joint work with Jan Baldeaux (UTS, Sydney) and with Josef Dick (UNSW, Sydney). Our findings are presented in full detail in the following two references:

References

[1] J. Baldeaux, M. Gnewuch. Optimal randomized multilevel algorithms for infinite-dimensional integration on function spaces with ANOVA-type decomposition. arXiv:1209.0882v1 [math.NA], Preprint 2012. (http://arxiv.org/abs/1209.0882)

[2] J. Dick, M. Gnewuch. Optimal randomized changing dimension algorithms for infinite-dimensional integration on function spaces with ANOVA-type decomposition, arXiv:1306.2821v1 [math.NA], Preprint 2013. (http://arxiv.org/abs/1306.2821)

Friday, 11.10 - 12.00

Randomized complexity of parametric problems

Stefan Heinrich
University of Kaiserslautern

We present a general scheme of solving parameter-dependent numerical problems in classes of smooth functions by randomized methods. The algorithm is a multilevel Monte Carlo procedure. We demonstrate this general approach by applying it to parametric definite and indefinite integration as well as to parameter-dependent initial value problems for ordinary differential equations.

Rates of convergence are obtained together with matching lower bounds. This way we determine the order of the randomized n-th minimal errors (in some cases up to logarithms), thus establishing the complexity of these problems. We also analyze the deterministic setting and give comparisons.

The problems are considered in anisotropic classes of functions, including certain classes with dominating mixed derivatives.
This is joint work with Th. Daun.

Em Höttche

Das historische Gasthaus am Rathaus
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GROUP CARD

„Rheinischer Sauerbraten“ 15,35 €
meat soaked in vinegar and stewed in
almond-raisin-sauce with potatoes-dumplings

„Himmel un Äad“ 12,90 €
calls sky and earth, fried blood pudding,
with liver sausage toped on mashed potatoes
and apple sauce
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sausage plate 14,50 €
with fried sausages, bologna sausage,
blood pudding, “Nürnberger” fried sausage,
sauerkraut and mashed potatoes
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beef liver “a la Berlin” with stewed onions, 13,65 €
apple chips, mashed potatoe and mixed salad
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chicken breast filled with leafty spinach, 12,60 €
sauce hollandaise, buttered rice and mixed salad
---
pork escalope *„house style“* 14,85 €
with braise onions, fried potatoes and mixed salad
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pork escalope *“vienna style”* 14,70 €
with fried potatoes, cranberry and mixed salad

„farmers pan“ 17,90 €
pork-fillet with bacon, grilled tomato, mushrooms
in cream sauce, fried potatoes and mixed salad

„HÖTTCHE`s vegetable-pan“ 12,35 €
baked cauliflower, brokoli, mushroom-cream-
sauce, toped with cheese and one potatoe pancake

„fitness – salad“ 13,25 €
mixed salad with fried turkey, fresh mushrooms,
peaws of peach, almond splitter, baguette and butter
