Karl-Theodor Sturm

Academic career

1989	Dr. rer. nat., University of Erlangen-Nürnberg
1989 - 1997	Postdoc, Universities of Zürich (Switzerland), Erlangen-Nürnberg, Bonn; Max Planck Institute, Leipzig
1993	Habilitation, University of Erlangen-Nürnberg
Since 1997	Professor (C3/W2/W3), University of Bonn

Honours

1991	Habilitationsstipendium (DFG)
1994	Heisenberg-Stipendium
2005	Professor Invité, Toulouse III, France
2008	Professor Invité, Paris VI, France
2016	ERC Advanced Grant “Metric measure spaces and Ricci curvature – analytic, geometric, and probabilistic challenges”
2017	“Hirzebruch Professor”, Max Planck Institute for Mathematics, Bonn

Offers

2006	Chair Professor, Imperial College, UK
2015	Distinguished Professor, Kansas University, USA
2015	Full Professor, PennState University, USA
2015	Full Professor, Northwestern University, USA

Invited Lectures

2009	Plenary speaker, Annual Meeting of the German Mathematical Society (DMV)
2010	Plenary speaker, 34th Conference on Stochastic Processes and Their Applications, Osaka, Japan
2011/12	Within one year, 6 lecture series (Imperial, SISSA Trieste, SMS Montreal, TopMath Munich, Midwest Probability Evanston, UK Easter Warwick)
2012	International conference dedicated to the centenary of L. V. Kantorovich, St. Petersburg, Russia
2014	Fields Medal Symposium in honour of C. Villani, Toronto, ON, Canada
2015	International conference dedicated to the centenary of K. Ito, Kyoto, Japan

Research Projects and Activities

DFG Cluster of Excellence “Hausdorff Center for Mathematics”
Member of the Board of Directors, since 2009, and coordinator, since 2012
DFG Collaborative Research Center SFB 1060 “The Mathematics of Emergent Effects” (projects B3, C1, C5)
Principal investigator and member of the steering committee, since 2013
DFG Collaborative Research Center SFB 611 “Singular Phenomena and Scaling in Mathematical Models” (projects A1, A2, A5, A9)
PI, vice director and member of the steering committee, 2002 - 2012
DFG Priority Program “Interacting stochastic systems of high complexity”
Principal investigator, 1997 - 2003
ERC Advanced Investigator Grant “Metric measure spaces and Ricci curvature - analytic, geometric, and probabilistic challenges”
Organizer of more than 15 International Conferences and Workshops within the last decade, among them e.g.
Trimester Program “Complex Stochastic Systems: Discrete vs. Continuous” at the Hausdorff

Research Institute HIM (Bonn) with 5 embedded workshops and 9 lecture series, 2007 - 2008
Workshops at IPAM (Los Angeles), CIRM (Luminy/Marseille), RIMS (Kyoto), Lebesgue Center (Rennes), CRM (Pisa), 2008 – 2013
Ongoing series of Oberwolfach Workshops on “Heat Kernels, Stochastic Processes and Functional Inequalities”, corresponding organizer, 2005, 2013, 2016
International Conference on Stochastic Analysis and Applications (SFB 611, Bonn)
– more than 250 registered participants, 2011
International Conference “New Trends in Optimal Transport” (SFB 1060, Bonn, 2015)
Conference “Panorama of Mathematics” (Bonn),
Organizer, 2015
Intense activity period on “Metric measure spaces and Ricci bounds”, 4 weeks of lecture series and invited/contributed talks at MPIM Bonn, 2017

Research profile

My research addresses a broad variety of problems from analysis, geometry and probability. Particular foci in recent years had been questions of optimal transport and effects of curvature in various contexts, including Riemannian manifolds, Finsler spaces, and infinite dimensional spaces (like path spaces, loop spaces, configuration spaces, or spaces of measures). With foundational publications I contributed to establish and to promote a new hot area in mathematics: metric measure spaces with synthetic Ricci bounds. Other research topics are matching problems, random measures, interacting particle systems and their scaling limits, gradient flows, geometric and functional inequalities, stochastic calculus on manifolds.
Major research directions for the future include: precise geometric structure of metric measures spaces with curvature-dimension condition (singular/regular points, boundaries) and their relation to Cheeger-Colding limits and Alexandrov spaces, analysis on time-dependent mm-spaces and their evolutions (e.g. under generalizations of Ricci flow), extension of ‘curvature’ concepts to spaces which so far had been out of reach (including branching or non-convex boundary conditions), stochastic calculus on mm-spaces, higher dimensional random geometries.

Research Area G One of the central ongoing topics of my research is the study of (deterministic or random) evolutions in environments of complex geometric structure. The evolutions under consideration might describe functions (e.g. states, particle densities), interacting particle systems, maps, or spaces. The complexity of the environment is due to irregularity, infinite dimensionality, or randomness. A major focus of my research in recent years is on optimal transport and its many applications, in particular, the interpretation of heat flows and other dissipative evolutions as gradient flows of entropy-like functionals on the Wasserstein space. Convexity properties of these functionals yield rates for equilibration and concentration, in many cases with explicit formulas for modifying these rates under tensorization, limits, conformal transformations, or time changes. Coupling of stochastic processes and optimal transport, leads to new deep insights in the study of time-dependent spaces e.g. evolving under (super-) Ricci flow. Important breakthroughs have been achieved in constructing optimal couplings between random measures, e.g. between point processes on $\mathbb{R}^n$.

Research Area A The topics of my research which in recent years attracted most international attention and publicity are the synthetic Ricci bounds for metric measure spaces and the far reaching geometry and geometric analysis developed on these spaces.
With two publications [8] and [9], - together with Lott and Villani (2009) who independently presented a similar concept - I laid the foundations to this innovative, flourishing field with many fascinating applications and stimulating interactions. In subsequent years, many further insights and results had been added, among them Ricci bounds for constructions (cones, suspensions), rough curvature bounds for discrete spaces, local-to-global property, essential non-branching, and existence of optimal maps.
As a landmark result, [3] established the equivalence of the ‘entropic’ curvature-dimension condition - defined in terms of optimal transport - with the celebrated ‘energetic’ curvature-dimension condition (or generalized ‘Bochner’s inequality’) introduced already 30 years ago by Bakry-Emery in terms of the so-called Γ calculus for diffusion operators. Besides this complete

equivalence of the Eulerian and the Lagrangian approach to heat flow and regularity issues on mm-spaces, various new estimates (space-time gradient estimate, Wasserstein control, N-log Sobolev inequality) had been derived in [3]. And the paper opened the door for many new developments and initiated various innovative research directions.

Supervised theses

Diplom theses: 35
PhD theses: 13, currently 3

Selected publications

[1] Janna Lierl and Karl-Theodor Sturm. Neumann heat flow and gradient flow for the entropy on non-convex domains. *eprint arXiv:1704.04164, to appear in Calc.Var.PDE*, 2017.

[2] Eva Kopfer and Karl-Theodor Sturm. Heat flows on time-dependent metric measure spaces and super-ricci flows. *eprint arXiv:1611.02570, accepted for Comm. Pure Appl. Math*, 2016.

[3] Matthias Erbar, Kazumasa Kuwada, and Karl-Theodor Sturm. On the equivalence of the entropic curvature-dimension condition and bochner's inequality on metric measure spaces. *Invent. Math.*, 201(3):993–1071, 2015.

[4] Shin-Ichi Ohta and Karl-Theodor Sturm. Bochner-weitzenböck formula and li-yau estimates on finsler manifolds. *Adv. Math.*, 252:429–448, 2014.

[5] Karl-Theodor Sturm. Gradient flows for semiconvex functions on metric measure spaces - existence, uniqueness and lipschitz continuity. *eprint arXiv:1410.3966, to appear in Proc. AMS*, 2014.

[6] Martin Huesmann and Karl-Theodor Sturm. Optimal transport from lebesgue to poisson. *Ann. Probab.*, 41(4):2426–2478, 2013.

[7] Karl-Theodor Sturm and Max-K. von Renesse. Entropic measure and wasserstein diffusion. *Ann. Probab.*, 37(3):1114–1191, 2009.

[8] Karl-Theodor Sturm. On the geometry of metric measure spaces. i. *Acta Math.*, 196(1):65–131, 2006.

[9] Karl-Theodor Sturm. On the geometry of metric measure spaces. ii. *Acta Math.*, 196(1):133–177, 2006.

[10] Karl-Theodor Sturm and Max-K. von Renesse. Transport inequalities, gradient estimates, entropy, and ricci curvature. *Comm. Pure Appl. Math.*, 58(7):923–940, 2005.