Matthias Lesch

Academic career

1988	Dr. rer. nat., University of Marburg (advisor: Manfred Breuer)
1989 - 1995	Assistant Professor (C1, later C2), University of Augsburg
1994	Habilitation, University of Augsburg
1994 - 1995	Visiting Assistant Professor, The Ohio State University, Columbus, OH, USA (on leave from Augsburg)
1995 - 1999	Senior Assistant Professor (C2), HU Berlin
1999	Heisenberg Fellow, University of Bonn
1999 - 2000	Associate Professor, University of Arizona, Tucson, AZ, USA
2001 - 2005	Professor (C3), University of Cologne
2005 - 2007	Professor (C3), University of Bonn
Since 2007	Professor (W2), University of Bonn

Honours

1995	Gerhard-Hess Award, German Research Foundation (DFG)
1999	Heisenberg fellowship, German Research Foundation (DFG)
2000	NSF Grant DMS 0072551

Offers

2007	Chair in Pure Math, Loughborough University, England, UK

Research profile

A large part of my research focuses around geometric differential operators (Dirac and Laplace operators) and their spectral theory. In particular I am interested in spectral invariants which may be extracted from the heat kernel (torsion, eta invariants, zeta-determinants, rho invariants).
I am also interested in Noncommutative Geometry a la Connes and I am working on heat invariants in the noncommutative setting. The noncommuative setting exhibits interesting phenomena which are generally not present in the commutative context.
In collaboration with Boris Vertman I have established a work programme on ”spectral geometry, index theory and geometric flows” in the context singular spaces. One of the main objectives is to establish a heat resp. resolvent expansion for certain Laplace type operators on certain stratified spaces. This would have interesting applications for the understanding of various of the above mentioned spetral invariants.
A second long term project is in operator algebras: I am working on functional analytic problems related to the construction of the celebrated Kasparov product in the unbounded picture of KK-theory.

Research Area A One of the main technical tools in spectral geometry is the asymptotic expansion of the heat trace of an elliptic operator. In [16], we study the Laplacian on singular algebraic curves and prove a complete asymptotic expansion of the heat trace. [15] studies regularity, Fredholmness and the heat expansion for general Fuchs type differential operators. These are the natural operator occuring in the context of conical singularities.
A striking new development is the discovery of Connes and Moscovici that the second heat coefficient of the Laplacian on the noncommutative torus exhibits universal one and two variable

functions with deep, not yet fully understood connections, to classical special functions. In [3] this is worked out for the Laplacian on all vector bundles (Heisenberg modules) over the noncommutative torus. More combinatorial aspects and an explanation of the universal functions in terms of divided differences is the content of [2].
Several other papers deal with more rigid spectral invariants (e.g. zeta-determinants, eta-invariants). In [1, 5, 11], zeta-determinants are calculated in singular one-dimensional situations motivated model operators occuring in the context of conical or hyperbolic singularities. This is motivated by the problem of extending the celebrated Cheeger-Müller Theorem on the equality of the analytic and combinatorial torsion to manifolds with singularities. The calculation of determinants is an important case study.

Research Area C Though I would consider myself a mathematician, I am dealing with mathematical structures which are of some relevance in Mathematical Physics:
KK-theory and K-theory play a prominent role in recently developed mathematical models of topological insulators. The main feature of Kasparov's KK-theory is the intersection product (aka Kasparov product), which also plays a prominent role in the bulk-edge correspondence of topological insulators. The Kasparov product is of intimidating generality and its construction is intimidating as well. In the last years I have worked intensely on various aspects of the unbounded picture of the Kasparov product [6, 9]. Currently, I am working with Bram Mesland on a constructive version of the intersection product in the unbounded picture, building on recent work by Mesland-Rennie and Brain-Mesland-van Suijlekom. [6, 9] were written while J. Kaad was a one year HCM Postdoc (Research Area C, mentor: M. Lesch) at the Mathematical Institute.
Structural questions about algebras of pseudodifferential operators are somewhat on the borderline between areas A and C. Differential operators embed nicely into an algebra of pseudodifferential operators. This is an algebra which contains also the parametrices of elliptic operators. This algebra has interesting structural properties. E.g. it has a unique trace which is sometimes called the Wodzicki-Guillemin residue trace and which plays an important role in renormalization theory, noncommutative geometry and in the asymptotic analysis of heat and resolvent traces. In [14], the residue trace is generalized to a larger class of pseudodifferential operators (log-polyhomogeneous), also it is shown that such operators do have a heat trace asymptotics where log t-powers occur. [8], essentially the PhD thesis of C. Neira Jimenez, gives a refined classification of residue traces trace functionals on subalgebras of the algebra of classical pseudodifferential operators.
The paper [11] is in a sense an amalgamation of my interest in the heat trace on the one hand and my fascination for Connes Noncommutative Geometry on the other hand. The Chern character of a spectral triple (in a very precise sense it is a generalization of the classical Chern character) in the so-called JLO version is defined in terms of (a generalization of) the heat trace. Its limits as t to 0 and t to infinity contain interesting spectral and geometric information. In the above mentioned paper we give a detailed account of this Chern character for a manifold with boundary and we calculate its limits.
The papers [3, 2], discussed under research area A, also belong to the context of noncommutative geometry and are therefore at the borderline between A and C.

Supervised theses

Master theses: 5, currently 3
Diplom theses: 11
PhD theses: 8, currently 2

Selected PhD students

Boris Vertman (2008): "The Analytic Torsion for Manifolds with Boundary and Conical Singularities",
now Professor, University of Münster
Carolina Neira Jimenéz (2010): "Cohomology Of Classes Of Symbols And Classification Of Traces On Corresponding Classes Of Operators With Non Positive Order",

now Assistant Professor, National University of Colombia
Batu Güneysu (2011): “On the Feyman-Kac formula for Schrödinger semigroups on vector bundles”,
now Postdoc, HU Berlin

Selected publications

[1] Luiz Hartmann, Matthias Lesch, and Boris Vertman. Zeta-determinants of sturm-liouville operators with quadratic potentials at infinity. *J. Differential Equations*, 262(5):3431–3465, 2017.
[2] Matthias Lesch. Divided differences in noncommutative geometry: rearrangement lemma, functional calculus and expansional formula. *J. Noncommut. Geom.*, 11(1):193–223, 2017.
[3] Matthias Lesch and Henri Moscovici. Modular curvature and morita equivalence. *Geom. Funct. Anal.*, 26(3):818–873, 2016.
[4] Alexander Gorokhovsky and Matthias Lesch. On the spectral flow for dirac operators with local boundary conditions. *Int. Math. Res. Not. IMRN*, (17):8036–8051, 2015.
[5] Matthias Lesch and Boris Vertman. Regularizing infinite sums of zeta-determinants. *Math. Ann.*, 361(3-4):835–862, 2015.
[6] Jens Kaad and Matthias Lesch. Spectral flow and the unbounded kasparov product. *Adv. Math.*, 248:495–530, 2013.
[7] Matthias Lesch. A gluing formula for the analytic torsion on singular spaces. *Anal. PDE*, 6(1):221–256, 2013.
[8] Matthias Lesch and Carolina Neira Jim’enez. Classification of traces and hypertraces on spaces of classical pseudodifferential operators. *J. Noncommut. Geom.*, 7(2):457–498, 2013.
[9] Jens Kaad and Matthias Lesch. A local global principle for regular operators in hilbert C^*-modules. *J. Funct. Anal.*, 262(10):4540–4569, 2012.
[10] Matthias Lesch, Henri Moscovici, and Markus J. Pflaum. Connes-chern character for manifolds with boundary and eta cochains. *Mem. Amer. Math. Soc.*, 220(1036):viii+92, 2012.
[11] Matthias Lesch and Boris Vertman. Regular singular sturm-liouville operators and their zeta-determinants. *J. Funct. Anal.*, 261(2):408–450, 2011.
[12] Paul Kirk and Matthias Lesch. The η-invariant, maslov index, and spectral flow for dirac-type operators on manifolds with boundary. *Forum Math.*, 16(4):553–629, 2004.
[13] Jochen Brüning and Matthias Lesch. On the η-invariant of certain nonlocal boundary value problems. *Duke Math. J.*, 96(2):425–468, 1999.
[14] Matthias Lesch. On the noncommutative residue for pseudodifferential operators with log-polyhomogeneous symbols. *Ann. Global Anal. Geom.*, 17(2):151–187, 1999.
[15] Matthias Lesch. *Operators of Fuchs type, conical singularities, and asymptotic methods*, volume 136 of *Teubner-Texte zur Mathematik [Teubner Texts in Mathematics]*. B. G. Teubner Verlagsgesellschaft mbH, Stuttgart, 1997.
[16] Jochen Brüning and Matthias Lesch. On the spectral geometry of algebraic curves. *J. Reine Angew. Math.*, 474:25–66, 1996.